
ASTRONOMICAL NAVIGATION MADE EASY

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PREFACE

ASTRONOMICAL Navigation is a subject which, owing to the amount of mystery with which the usual textbooks surround it, appears to present untold difficulties to those who have to learn something about it.

As a practical air pilot and navigator, the Author fully realizes the needs of fellow pilots—and others—whose time is too valuable to be spent in hours of search, and in this little book has tried to set out in an "easy reference" manner just the essential data required. He has seen the subject from the point of view of the novice, and with this in mind adds, as an appendix, the fully explained method of application of Plane Trigonometry and Logarithms; this knowledge being an essential requirement before any attempt is made to deal with Astronomical Navigation or Spherical Trigonometry.

It is also necessary to possess, carefully study, and use *Norie's Nautical Tables*, a current abridged edition of the *Nautical Almanac*, and a sextant—of either the marine or the bubble type. The intricacies of a sextant are more easily understood if it can be handled and worked with on one's own, and in the early stages of study any cheap, secondhand, marine-type sextant will do. It may not be accurate, but strict accuracy is non-essential at this stage. In the same way, an ordinary wrist watch is sufficiently accurate for timing one's initial efforts.

No attempt has been made to explain the "why and wherefore"; this is already available in numerous books on the subject, perhaps the best of which is *The Admiralty Manual of Navigation*, Vol. I. What has been aimed at is the practical application in a concise form of a mass of detail for the help of those who need to learn Astronomical Navigation for use by sea or air.

G. W. F.

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1937

CONTENTS

ASTRONOMICAL NAVIGATION MADE EASY

SPHERICAL TRIGONOMETRY

BEFORE astronomical navigation problems can be solved, it is necessary to know something about spherical trigonometry. In dealing with spherical trigonometrical solutions there is so much wangling necessary that the following facts should be learnt by heart so that they may be applied at any time.

1. The Complement of an angle is the angle subtracted from 90°.
2. The Supplement of an angle is the angle subtracted from 180°.
3. The Complements of angle A are as follows—

$$\begin{aligned}
\text{Sin } A &= \text{Cos } (90° - A)\\
\text{Cos } A &= \text{Sin } (90° - A)\\
\text{Tan } A &= \text{Cot } (90° - A)\\
\text{Cot } A &= \text{Tan } (90° - A)\\
\text{Cosec } A &= \text{Sec } (90° - A)\\
\text{Sec } A &= \text{Cosec } (90° - A)
\end{aligned}$$

If the latitude is then known, then the complement of the latitude = 90° − Latitude. This is the Co-Lat. Therefore Sin Lat. = Cos Co-Lat. If the declination is known and the latitude and declination are the same name, i.e. both north or both south, then the complement of the declination = 90° − Declination. This is the Polar Distance. The Polar Distance could also be 90° + Declination and would be so if the observer is in the northern hemisphere and a star has declination south. There is no complement for angles over 90°.

If the altitude of a body is known, then the complement of the altitude = 90° − Altitude. This is the Co-Alt. or Zenith Distance.

4. *To Make Logs Additive.* It often happens that a division sum has to be done when dealing with trig. ratios and, as it is usually easier to add rather than to subtract the logs. in doing such a sum,

it is convenient to change the denominator into a numerator or top figure. In doing this

$$\frac{1}{\text{Sin } A} \text{ becomes } 1 \times \text{Cosec } A \qquad \frac{1}{\text{Cot } A} \text{ becomes } 1 \times \text{Tan } A$$

$$\frac{1}{\text{Cos } A} \quad \text{,,} \quad 1 \times \text{Sec } A \qquad \frac{1}{\text{Sec } A} \quad \text{,,} \quad 1 \times \text{Cos } A$$

$$\frac{1}{\text{Tan } A} \quad \text{,,} \quad 1 \times \text{Cot } A \qquad \frac{1}{\text{Cosec } A} \quad \text{,,} \quad 1 \times \text{Sin } A$$

Thus $\frac{\text{Sin } A}{\text{Cos } B}$ could be written Sin A × Sec B

and $\frac{\text{Tan } A}{\text{Cot } B}$,, ,, ,, Tan A × Tan B.

5. *Angles more than* 90°. When an angle is more than 90° it will not be possible to find it direct from the tables. It must be subtracted from 180°. Thus if Log Cosec $101\frac{1}{2}°$ is needed, find the Log Cosec of $180° - 101\frac{1}{2}° =$ Log Cosec $78\frac{1}{2}°$.

RIGHT-ANGLED SPHERICAL TRIANGLES

In a spherical triangle where one of the angles is 90°, the triangle is a right-angled triangle. In order to solve these types of triangles, such as that illustrated in Fig. 1, draw a rough circle (Fig. 2) and put a dot in the middle. Draw a vertical line from the dot to the bottom of the circle, and then divide the circle into five parts. In the three opposite, but not in the two adjacent segments to the vertical line, write the letters "co." Starting from the right angle of the triangle, go round the triangle in sequence and label each segment from the vertical line. It does not matter which way round you go, but you must start at the right angle of the triangle and at the vertical line in the circle.

Napier's Rules for Right-angled Spherical Triangles with Circle of Five Parts

Sin Middle Part = Product of Tangent of Adjacent parts or
Product of Cosine of Opposite parts

subject to the following sub-rules—

1. The side taken as middle changes from Sine to Cosine if it has a "co."

2. The adjacent sides are normally Tangent, but if they have a "co" they turn Tangent into Co-Tangent.

3. The opposite sides are normally Cosine, but if they have a 'co" they turn Cosine into Sine.

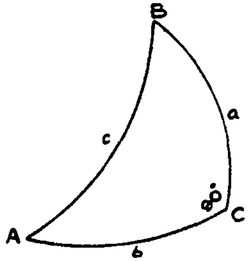


FIG. 1

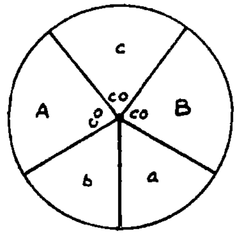


FIG. 2

b middle.	$\text{Sin } b$	$= \text{Cot } A \text{ Tan } a$	or	$\text{Sin } c \text{ Sin } B$
c ,,	$\text{Cos } c$	$= \text{Cot } A \text{ Cot } B$	,,	$\text{Cos } b \text{ Cos } a$
A ,,	$\text{Cos } A$	$= \text{Cot } c \text{ Tan } b$	,,	$\text{Sin } B \text{ Cos } a$
B ,,	$\text{Cos } B$	$= \text{Tan } a \text{ Cot } c$	,,	$\text{Sin } A \text{ Cos } b$
a ,,	$\text{Sin } a$	$= \text{Tan } b \text{ Cot } B$	,,	$\text{Sin } A \text{ Sin } c$

To know if the side or angle is as calculated or the supplement (180° — calculated angle) apply the following rules—

1. The legs, i.e. base and perpendicular (not the hypotenuse), are of the same affection as their opposite angles. This means that if the angle at A (Fig. 1) is less than 90°, then the leg CB is less than 90° and vice versa.

2. Every side over 90° is greater than the hypotenuse, but every side less than 90° is less than the hypotenuse.

3. The hypotenuse is less than 90° if

(*a*) the other two sides are of like affection,
or (*b*) the two angles other than the 90° angle are of like affection.

4. The hypotenuse is more than 90° if

(*a*) the other two sides are of unlike affection,
or (*b*) the two angles other than the 90° angle are of unlike affection.

Thus, if the hypotenuse is more than 90°, the other two sides, and their opposite angles, must be of unlike affection, and if one is known to be less than 90°, then the other must be more than 90°, and so must be more than the hypotenuse, *vide* item (2).

Quadrantal Spherical Triangles

Quadrantal spherical triangles are triangles where one or more sides are 90° of arc.

Napier's rules with the circle of five parts apply. Treat the quadrantal side as the starting point, and for the vertical line in drawing the lines inside the circle.

Special Rules

(*a*) The sides other than the quadrantal side are of the same affection as that of the angle to which they are opposite.

(*b*) The angle opposite the quadrantal side is greater than 90° when the other two angles or sides are of like affection, but less than 90° when the other two sides or angles are of unlike affection. This is the reverse to the rule for right-angled spherical triangles.

Oblique-angled Spherical Triangles

Where no side or angle is 90°, the triangle is an oblique-angled triangle. In these types of triangles, the Sine Formula may sometimes be used to solve part of it, but, in certain cases, there may be two solutions and the results are therefore ambiguous. To find if there is an ambiguous case apply the following rules—

1. *Given two sides and an angle opposite one*—

If the value of the side opposite the given angle lies between the value of the other given side and the supplement of this other given side, there will be only one solution.

2. *Given two angles and a side opposite one*—

If the value of the angle opposite the given side lies between the value of the other given angle and the supplement of this other given angle, there will be only one solution.

The Sine Formula is common to all forms of oblique-angled spherical triangles and is as follows—

$$\frac{\operatorname{Sin} A}{\operatorname{Sin} a} = \frac{\operatorname{Sin} B}{\operatorname{Sin} b} = \frac{\operatorname{Sin} C}{\operatorname{Sin} c}$$

where A, B, and C are the angles and a, b, and c are the sides opposite these angles.

Oblique-angled Spherical Triangles. Solutions

Recommended

1. *Given three sides*—

Use Haversine Formula.

2. *Given two sides and included angle*—

Find other side by Haversine Formula. Find other angles by Sine Formula, or by Haversine Formula.

3. *Given two sides and angle opposite one of them*—

Find angle opposite the other given side by Sine Formula.

Find other side by dividing triangle into two right-angled triangles.

Find other angle by Sine Formula, or by Haversine Formula.

4. *Given two angles and a side opposite one of them*—

Find side opposite the other given angle by Sine Formula.

Find other side by dividing the triangle into two right-angled triangles.

Find other angle by Sine Formula.

5. *Given two angles and included side*—

Has its own formula.

6. *Given three angles*—

Has its own formula.

Important Note. When the Sine Formula is used, if the angles or sides are small, the results are not strictly accurate. In these cases it is better to use the Haversine Formula. Also, the angle or arc given by the Sine Formula may not be the actual angle, the supplement being the actual angle. It is often difficult to know which is the correct angle. The Haversine Formula leaves no doubt.

Haversine Formula

Hav. a = Hav. $(b \sim c)$ + Sin b Sin c Hav. A.

A is the included angle, b and c the other two sides.

If it is desired to find the other side, the work would be set out as under.

Log Hav. A	=	....
*Log Sin b	=	....
*Log Sin c	=	....
Sum. Log Hav. θ		
Nat. Hav. θ		
$b \sim c$ Nat. Hav.		
Sum. Nat. Hav.		....

a = ... from Nat. Hav.

* If using the complement, viz. $(90° - b)$ and $(90° - c)$, the Cosine, and not the Sine, is used. Such will be the case in working out the C.Z.D. (Calculated Zenith Distance). Log Haversines and Natural Haversines are given in *Norie's Nautical Tables.*

Haversine Formula

Given Three Sides, to Find Angles

To find angle A—

Nat. Hav. θ = Nat. Hav. a − Nat. Hav. $(b \sim c)$.
Log Hav. A = Log Cosec b + Log Cosec c + Log Hav. θ.

To find angle B—

Nat. Hav. θ = Nat. Hav. b − Nat. Hav. $(c \sim a)$.
Log Hav. B = Log Cosec c + Log Cosec a + Log Hav. θ.

To find angle C——

Nat. Hav. θ = Nat. Hav. c − Nat. Hav. $(a \sim b)$.
Log Hav. C = Log Cosec a + Log Cosec b + Log Hav. θ.

Haversine Formula

Given Two Sides and the Included Angle

To find side

Log Hav. θ = Log Hav. A + Log Sin b + Log Sin c.
Nat. Hav. a = Nat. Hav. θ + Nat. Hav. $(b \sim c)$.

Note. This is exactly the same formula as that to find angle A given three sides, but twisted round in a different way.

To find side b—

Log Hav. θ = Log Hav. B + Log Sin c + Log Sin a.
Nat. Hav. b = Nat. Hav. θ + Nat. Hav. $(c \sim a)$.

To find side c—

Log Hav. θ = Log Hav. C + Log Sin a + Log Sin b.
Nat. Hav. c = Nat. Hav. θ + Nat. Hav. $(a \sim b)$.

Find the other angles by the Sine Formula, or by Haversine Formula for Three Sides Given.

Given Two Sides and an Angle Opposite One of them

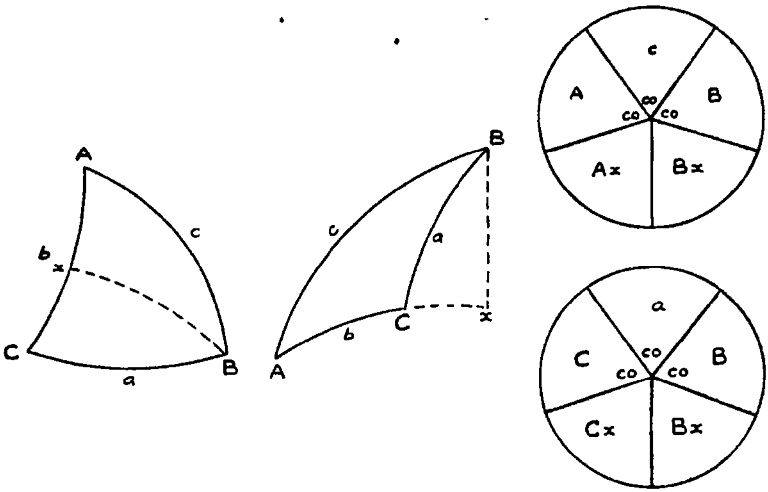


Fig. 3

Given a, c, and A.

1. Find angle opposite the other given side by the Sine Formula.

$$\frac{\text{Sin } C}{\text{Sin } c} = \frac{\text{Sin } A}{\text{Sin } a}$$

$$\therefore \quad \text{Sin } C = \text{Sin } A \text{ Sin } c \text{ Cosec } a$$

$$\therefore \quad C = \ldots\ldots\ldots$$

2. (i)

$$\text{Cos } A = \text{Tan } Ax \text{ Cot } c$$

$$\therefore \quad \text{Tan } Ax = \text{Cos } A \text{ Tan } c$$

$$\therefore \quad Ax = \ldots\ldots\ldots$$

(ii)

$$\text{Cos } c = \text{Cos } Ax \ \boxed{\text{Cos } Bx}$$

$$\text{and} \quad \text{Cos } a = \text{Cos } Cx \ \boxed{\text{Cos } Bx}$$

$$\therefore \quad \frac{\text{Cos } c}{\text{Cos } Ax} = \frac{\text{Cos } a}{\text{Cos } Cx}$$

$$\therefore \quad \text{Cos } Cx = \text{Cos } a \text{ Cos } Ax \text{ Sec } c$$

$$\therefore \quad Cx = \ldots\ldots\ldots$$

Side $b = Ax + Cx$ if x falls inside the triangle, but $Ax - Cx$ if x falls outside the triangle.

In words, this becomes,

Cos of given side : Cos of part already found

as Cos of other given side : Cos of other portion of part to be found.

3. To find other angle B, this may be found by the Sine Formula or by the Haversine Formula.

Given Two Angles and a Side Opposite One of Them

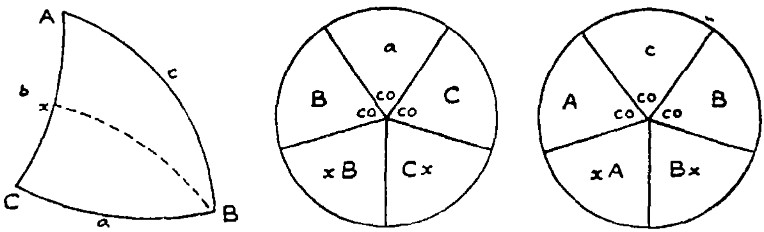


Fig. 4

Given A, C, and a

1. Find the side opposite the other given angle by the Sine Formula.

$$\frac{\text{Sin } c}{\text{Sin } C} = \frac{\text{Sin } a}{\text{Sin } A}$$

$$\therefore \quad \text{Sin } c = \text{Sin } a \text{ Sin } C \text{ Cosec } A$$

$$\therefore \quad c = \ldots\ldots\ldots$$

2. (i) $\text{Cos } C = \text{Tan } Cx \text{ Cot } a$

$$\therefore \quad \text{Tan } Cx = \text{Cos } C \text{ Tan } a$$

$$\therefore \quad Cx = \ldots\ldots\ldots$$

(ii) $\text{Sin } Cx = \text{Cot } C \; \boxed{\text{Tan } xB}$

and $\text{Sin } Ax = \text{Cot } A \; \boxed{\text{Tan } xB}$

$\therefore$ as $\text{Sin } Ax : \text{Cot } A :: \text{Sin } Cx : \text{Cot } C$

$$\therefore \quad \frac{\text{Sin } Ax}{\text{Cot } A} = \frac{\text{Sin } Cx}{\text{Cot } C}$$

$$\therefore \quad \text{Sin } Ax = \text{Sin } Cx \text{ Cot } A \text{ Tan } C$$

$$\therefore \quad Ax = \ldots\ldots\ldots$$

This may be put into words thus—

Sin of part already found : Cot of its adjacent angle
as Sin of other portion of part to be found : Cot of its adjacent angle

The side AC = the sum of Ax and Cx where x falls inside the triangle. Should x fall outside the triangle, the rules for solving the triangle apply, but the side to be found will not be the sum but the difference of the two parts found.

3. To find the remaining part, angle B, this may now be found by the Sine Formula, or Haversine Formula.

Given Two Angles and the Included Side

$$\text{Tan } \tfrac{1}{2}(a+b) = \frac{\text{Cos } \frac{1}{2}(A-B)}{\text{Cos } \frac{1}{2}(A+B)} \text{Tan } \frac{c}{2}$$

$$= \text{Cos } \tfrac{1}{2}(A-B) \text{ Sec } \tfrac{1}{2}(A+B) \text{ Tan } \frac{c}{2}$$

$$\text{Tan } \tfrac{1}{2}(a \sim b) = \frac{\text{Sin } \frac{1}{2}(A-B)}{\text{Sin } \frac{1}{2}(A+B)} \text{Tan } \frac{c}{2}$$

$$= \text{Sin } \tfrac{1}{2}(A-B) \text{ Cosec } \tfrac{1}{2}(A+B) \text{ Tan } \frac{c}{2}$$

$\frac{a+b}{2}$ and $\frac{a \sim b}{2}$ added gives greater side, and subtracted, lesser side.

If $\frac{1}{2}$ sum of sides is less than 90°, then $\frac{1}{2}$ sum of angles will also be less than 90°, but if $\frac{1}{2}$ sum of sides exceeds 90°, then $\frac{1}{2}$ sum of angles must be taken from 180°.

Find the other angle by the Sine Formula, or Haversine Formula.

Note. This formula is given for what it is worth. It will be found that results obtained by this formula are not always strictly accurate when one of the angles or the included side is of a small amount.

Given Three Angles

Find the supplement of one of the angles. Take the largest angle for preference (180° – largest angle). To this add the other two angles.

Then $\text{Cos } \frac{1}{2} b = \sqrt{\frac{\text{Sin } S \text{ Sin } R}{\text{Sin } A \text{ Sin } C}} = \sqrt{\text{Sin } S \text{ Sin } R \text{ Cosec } A \text{ Cosec } C}$

where b = one side to be found.

$S = \frac{1}{2}$ (Sum of two angles + supplement of the third).

$R = S -$ angle opposite required side, in this case b.

Having found one side, knowing the other angles the remaining sides may be found by the Sine Formula, or Haversine Formula.

Note. This formula is given for what it is worth. It will be found that the results are not strictly accurate when one of the angles is of a small amount in comparison with the other two.

FORMULA FOR AMPLITUDES

Sin Amplitude = Sin Declination × Sec Latitude

All bodies rise in the east and set in the west the world over. The Amplitude is N. or S. of E. or W. by the amount of the angle found by formula according to whether the declination is N. or S.

Example. What is the sun's true bearing at sunrise and sunset on 30–1–37, Latitude 51° 30′ N., Longitude all ranges?

The *Nautical Almanac* shows sun's declination to be 17° 41½′ S. at noon on 30–1–37.

Sin Amp. = Sin 17° 41½′ Sec 51° 30′

Log Sin 17° 41½′	=	9·482723
Log Sec 51° 30′	=	10·205850
		19·688573

∴ Amp. = 29° 13′

Sunrise E. 29° S. = 119°

Sunset W. 29° S. = 241°

Normally this would be obtained from the Amplitude Tables in *Norie's* without working it out by formula.

FORMULA FOR LATITUDE AND LONGITUDE OF VERTEX

Latitude of Vertex

Cos Lat. Vertex = Cos Lat. of A Sin Course at A.

Longitude of Vertex

Cot Long. from A = Sin Lat. of A Tan Course at A.

Where A in both cases is either the starting point or destination.

Note on Great Circle Courses. When a start has been made in the northern hemisphere the Course Angle is from north towards east if going farther east, and towards west if going farther west. In the

southern hemisphere the Course Angle is the angle from south towards east if going farther east, and towards west if going farther west.

The Great Circle course angle as found by means of the Sine Formula will be less than 90°, but this may not be the actual angle. To find if the actual angle is more than 90°, make a note of the latitude of the place of departure and place of destination. If the latter is nearer the equator than the place of departure, the angle will normally be more than 90°, in which case subtract the angle as found by formula from 180° in order to get the actual angle. The Haversine Formula gives the actual angle direct.

The Great Circle course is always on the Polar side of the Rhumb Line course. The angular difference between the initial G.C. course and the Rhumb Line course is given by the formula $\frac{1}{2}$ *d* Long. × Sin Mid. Lat.

The Course Angle for finding the Vertex is the Great Circle course angle and not the Rhumb Line course angle.

If the angles within the spherical triangle forming the initial Great Circle courses are both less or both more than 90°, the Vertex lies somewhere between the two places, but if one of the angles is more than 90° and the other less than 90°, the Vertex lies outside the two places.

Formula for Finding Latitude at Different Points along the Great Circle from the Meridian of the Vertex

Tan Lat. *p* = Cos diff. of Long. of *p* from Vertex Meridian multiplied by Tan Lat. of Vertex.

Where *p* = point away to side of Vertex for which the latitude is required.

Composite Great Circle Courses

Should it be desired to follow a Great Circle course in the main but at the same time not go so far north or so far south as the Vertex, then a composite G.C. course could be steered whereby the G.C. course is followed for a proportion of the route, and then the course is altered to follow a parallel of latitude for a while, thereafter reverting to the G.C. course.

Such a procedure would mean three primary separate calculations—

1. To find the latitude of the Vertex if the G.C. course had been steered from the start to the destination.
2. To find the initial G.C. course to steer from the start to the

highest latitude it is desired to go, which latitude will be nearer the equator than the latitude of the Vertex.

3. To find the initial G.C. course to steer from the point where the parallel sailing will revert to the G.C. course connecting the destination.

In the following formulae

s and d = Start and destination respectively.

P = Pole and longitude angle.

V' and V'' = Point of highest latitude it is desired to go, both of which points are on the same parallel of latitude.

Then Cos Long. V' from s = Tan Co-Lat. V' Cot Co-Lat. s

and
$$\text{Cos dist. } sV' = \frac{\text{Cos } Ps \text{ (Co-Lat. of } s)}{\text{Cos } PV' \text{ (Co-Lat. of } V')}$$

This fixes the position and distance of V' relative to s, knowing which the initial G.C. course angle from s to V' may be calculated by the Sine Formula, the following parts being known—

Side corresponding to distance s to V'.
,, Co-Lat. V'.
,, Co-Lat. s.
Angle difference of longitude of s relative to V'.

The next step is to fix the longitude of V''.

Cos Long. V'' from d = Tan Co-Lat. V'' Cot Co-Lat. d

and
$$\text{Cos dist. } dV'' = \frac{\text{Cos } Pd \text{ (Co-Lat. of } d)}{\text{Cos } PV'' \text{ (Co-Lat. of } V'')}$$

Knowing the difference of longitude between d and V'' the longitude of V'' is fixed, and as V'' and V' are on the same parallel of latitude, the next step is to find out the distance between the two places so as to know how long the course must be steered due east or west.

Dist. on same parallel = d Long. $\times$ Cos Lat.

The final stage is to calculate the initial G.C. course from V'' to d, which may be done by the Sine Formula in which the following parts are known—

Side corresponding to distance V'' to d.
,, Co-Lat. V''.
,, Co-Lat. d.
Angle difference of longitude of V'' relative to d.

To Calculate ? Use the Formulae Noted Below

1. *Altitude*

(i) Haversine Formula (two sides and included angle) modified for latitude and declination if desired.

or (ii) Bygrave Position Line Slide Rule, which gives altitude direct.

Parts known are—

Side Co-Lat.
,, Polar Distance.
Angle Hour Angle.

Part required—

Side opposite Hour Angle which is to be subtracted from 90° to obtain the calculated altitude, if Haversine Formula used.

2. *Amplitude*

Readily found from tables in *Norie's*. If calculated, use Amplitude Formula.

3. *C.Z.D.*

(i) Haversine Formula (two sides and included angle) modified for latitude and declination if desired.

or (ii) Bygrave Position Line Slide Rule, which gives altitude, and this must be subtracted from 90° to obtain C.Z.D.

Parts known are—

Side Co-Lat.
,, Polar Distance.
Angle Hour Angle.

Part required—

Side opposite Hour Angle which is the C.Z.D.

4. *Distance along a Parallel of Latitude*

Distance in ′ (nautical miles) = d/Long. in ′ × Cos Lat.

5. *Great Circle Courses*

Two stages. Parts known are—

Side Co-Lat.
,, Co-Lat.
Angle *d*/Long.

1st Stage. Find other side by Haversine Formula (two sides and included angle).

2nd Stage. Find other angles by Sine Formula, or Haversine Formula.

Note on Course Angle. To decide whether the course angle is as calculated, see the rules which have been dealt with under the heading "Formula for Latitude and Longitude of Vertex." (Page 11.)

6. *Great Circle Distances*

Haversine Formula (two sides and included angle) modified for latitude if required.

Parts known are—

Side Co-Lat.
,, Co-Lat.
Angle *d*/Long.

Part required—

Side opposite *d*/Long. Angle.

7. *Hour Angle*

For Longitude use Haversine Formula (three sides). See Longitude.

For Latitude use Sine Formula using parts given below—

Side Co-Alt. or Zenith Distance.
,, Polar Distance.
Angle Azimuth.

Part required—

Angle opposite Co-Altitude.

Hour Angle so found in both cases may have to be subtracted from 24 hrs. (See page 24.)

8. *Latitude*

(i) Two R.A. triangles (two sides and angle opposite one).

Parts known are—

Side Co-Alt. or Zenith Distance.
,, Polar Distance.
Angle Hour Angle.

Part required—

Third side which is Co-Lat. and which must be subtracted from 90° to give latitude.

or (ii) **Haversine Formula doctored for use only when sight has been taken near meridian altitude. See special application of formula, page 42.**

9. *Longitude*

Haversine Formula (three sides).

Parts known are—

Side Co-Alt. or Zenith Distance.
" Co-Lat.
" Polar Distance.

Part required—

Angle opposite side Co-Alt.

This gives the Hour Angle, knowing which the longitude may be found. (See worked example under longitude.) The Hour Angle so calculated above may have to be subtracted from 24 hrs. (See pages 21–24.)

10. *Rhumb Line Courses and Distances*

(i) Departure $= d/\text{Long. in}\ ' \times \text{Cos Mid-lat.}$

$$\text{Tan Course} = \frac{\text{Departure}}{d/\text{Lat. in}\ '}$$

$$\text{Distance} = d/\text{Lat. in}\ ' \times \text{Sec Course}$$

(ii) $$\text{Tan Course} = \frac{d/\text{Long. in}\ '}{d/\text{Meridional Parts}}$$

$$\text{Distance} = d/\text{Lat. in}\ ' \times \text{Sec Course}$$

Note. The Meridional Parts for the latitude of starting place and for latitude of destination are found in *Norie's Tables*. Subtract one from the other to get the difference.

The course found in the above cases will be 90° or less, and is to be reckoned in the nautical manner.

60° = N. 60° E.	300° = N. 60° W.
120° = S. 60° E.	240° = S. 60° W.

In order to ascertain the direction, see whether you are going more north or south. If you are going more south, write S. followed by the angle. Then see if you are going more east or more west and fill in this direction following the angle.

11. *To Convert Nautical Miles to Statute Miles*

Nautical to Statute: Add 15%.

Example:		164 nautical miles;
	10% =	16·4
	5% =	8·2 (Half of 10%)
		188·6 statute miles.

Statute to Nautical: Divide by 1·15.

12. *Vertex, Latitude of*

Vertex latitude formula.

13. *Vertex, Longitude of*

Vertex longitude formula.

BYGRAVE POSITION LINE SLIDE RULE

One of the great advantages of the Bygrave Slide Rule is that it gives the azimuth as well as the altitude and thence C.Z.D. (Calculated Zenith Distance)—

Data needed	*Data found*
Declination.	Azimuth.
Hour Angle in degrees.	Altitude and thence C.Z.D.
Co-Latitude.	(90° − Alt.) = C.Z.D.

Set pointers to Zero.
Turn Hex. head cylinder so that Dec. is opposite pointer L.
Turn milled ring and pointer S to Hour Angle.
Read value 'y' at pointer L.
Lat. and Dec. same name Co-Lat. $+ y = Y$.
,, ,, contrary Co-Lat. $\sim y = Y$.
Set S to y.
Turn Hex. head until Hour Angle at L.
Set S to Y.
Read azimuth at L.
Set S to azimuth.
Turn Hex. head so that L is set to value Y.
Set milled ring to zero.
Read altitude at L. L = Long pointer on instrument.
90° − Alt. = C.Z.D. S = Short pointer on instrument.

Azimuth is determined as regards N. or S. from pole of opposite latitude, and E. or W. according to whether the Hour Angle is east or west.

Definitions

Longitude is the angle measured at the pole between a meridian passing through a place and some prime meridian such as the Greenwich Meridian.

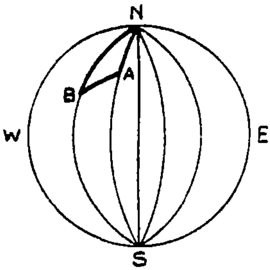


Fig. 5

If the line NAS was the Greenwich Meridian, the longitude of a place B would be the angle BNA, and as B is west of the Greenwich Meridian, its longitude would be west.

Meridian. A meridian is a line joining the North and South Poles on one side of the sphere. There can be an infinite number of meridians and they are not limited to those shown on a map. The line on the other side of the sphere is called an anti-meridian.

Latitude is the arc of a meridian intercepted between the equator and a small circle passing through a place. It is measured N. or S. of the equator towards the pole. In the diagram, $NBES$ is a meridian. WE the equator, and BE an arc of a meridian. AB is a small circle, in this case a parallel of latitude. Latitude is also an angle measured at the centre of the world from the equator N. or S. of the equator. 1′ of latitude on all maps and charts of whatever scale is one nautical mile. Thus 1° of latitude is 60 nautical miles. 1° of longitude is 60 nautical miles on the equator only. At the poles longitude has no distance.

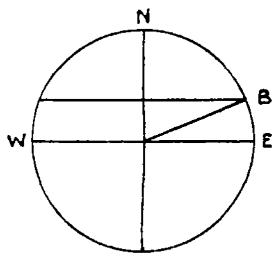


Fig. 6

Zenith. A point directly overhead. The observer's zenith is vertically directly above him. The sun's or other body's zenith position on the earth's surface is a point on the surface of the earth where the body is vertically directly overhead. The angle from the horizon to an observer's zenith is 90°.

Altitude of body is the angle or arc that the body is above the horizon or horizontal. This angle cannot exceed 90°.

It is not proposed to give any definition of the following as it is not necessary to know how they are constituted so far as the practical part of navigation is concerned. Their values are tabulated in the abridged edition of the *Nautical Almanac* for the month and the day, and often hour, against the body being dealt with and this book is intended only to show how they are applied.

Right Ascension	R.A.
Factor *R* for Sun	*R*.
Factor *E* for Sun	*E*.
Declination	Dec.

For the same reason it is not proposed to define—

Hour Angle	H.A.
Right Ascension of the Meridian . .	R.A.M.

General Diagram for Astronomical Problems

In dealing with astronomical problems, it often helps to draw a diagram in order to visualize more readily the position, and it is recommended that a diagram shall be drawn in all cases in the early stages of dealing with problems.

Diagram for the Northern Hemisphere

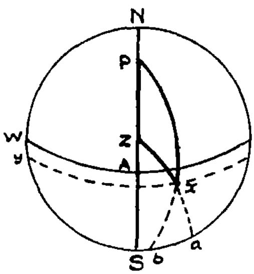


Fig. 7

To draw the diagram, make a circle. Draw a vertical line down the middle. This line is your meridian and you are at *Z* in the centre of the circle. Mark a spot *P* on your meridian to represent the North Pole, and another spot *A* so that the distance $NP = ZA$. Through *A* draw a curved line *WAE* across the circle. This is the equator line.

Then NP = Latitude—even if it does not look as if it does.
PZ = Co-Latitude ditto
ZA = Latitude ditto
WE = Equator.
Z = Your position and is your Zenith Position.

Having drawn in the lines so far, proceed to fill in the others. First find out from the *Nautical Almanac* whether the declination of the body being dealt with is north or south. If it is south, draw a curved

dotted line yy' on the south side of the equator line parallel to the equator line. This dotted line is the declination line. Had the declination been north, the dotted line would have been drawn on the north side of the equator line. If the body being dealt with is on the east side of your meridian, mark a spot x on the declination line on the eastern half of the circle. Join Px by a curved line and continue it in dotted form to the edge of the circle. Join Zx also by a curved line and continue it in dotted form to a.

Then Px = Polar Distance.
Zx = Co-Altitude or Zenith Distance.
xa = Altitude of body.
Angle PZx = Azimuth or bearing of body from north through east.
Angle ZPx = Hour Angle.

The spherical triangle so formed by the diagram is PxZ.

Similar Diagram for the Southern Hemisphere

Had the latitude been south, and the declination south, and Hour Angle east as for the northern hemisphere, the diagram could be drawn either way as illustrated in Fig. 8.

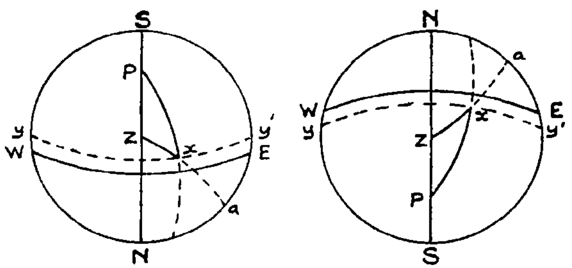


Fig. 8

POLAR DISTANCE

The Polar Distance of a body is the arc of the meridian between the zenith position of the observed body and the pole of the hemisphere in which you are situated. If the declination of the body is north and you are in the northern hemisphere, then the Polar Distance is 90° – Dec.

If the declination of the body is south and you are in the northern hemisphere, then the Polar Distance is 90° + Dec.

From the foregoing it will be observed that the body is always on the declination line. The amount of the declination for all bodies is given in the *Nautical Almanac*. The Polar Distance is thus a quantity which may be found at all times.

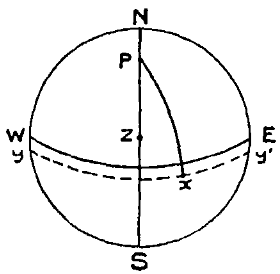


FIG. 9

WE = Equator.
P = Elevated Pole.
Z = Observer's zenith position.
yy' = Declination line.
x = Zenith position of the observed body on the declination line.
Px = Polar Distance.

TO FIND LONGITUDE FROM SUN WHEN NOT AT MERIDIAN ALTITUDE

Formula

Longitude = G.M.T. + E. – H.A. (All in 0 hrs.–24 hrs. notation.)
If result is less than 12 hrs., longitude is west.

If result is greater than 12 hrs., subtract from 24 hrs. to give east longitude.

If, after subtracting H.A., result is more than 24 hrs., deduct 24 hrs. before deciding whether longitude is east or west.

Data needed

1. G.M.T. and G.D. at place.
2. True Altitude of sun and approximate azimuth.
3. D.R. Latitude.
4. Declination of sun for G.M.T. and G.D.
5. E. for G.M.T. and G.D.

Working

First find the calculated Hour Angle. Use Haversine Formula (three sides known), *a*, Co-Altitude; *b*, Polar Distance; *c*, Co-Latitude. The H.A. (angle *A*) so found is east or west relative to the observer's meridian. Having found the H.A. by calculation, convert it into an H.A. in the 24 hrs. notation before applying it to the formula for finding longitude.

Note on Sun's Hour Angle. In most problems dealing with astronomical navigation the H.A. presents the greatest difficulty. For reasons which appear beyond comprehension, the sun's H.A. is 0 hrs. when the true sun is opposite the observer's meridian on its way towards the west. It is 12 hrs. when opposite the observer's anti-meridian at midnight, and 24 hrs. when back again opposite the meridian having passed through east. In this form the H.A. is the same in value as L.A.T. in the 24 hrs. system $\pm$ 12 hrs. It would have made things so much easier had the powers that be made the sun's H.A. the same as L.A.T.

Although the sun's H.A. is measured as 0 hrs.–24 hrs. from the observer's meridian, it may also be measured as an angle or time arc east or west of the observer's meridian. In using the Haversine Formula for calculations, or to work out a Position Line, it is advisable to measure the H.A. in this manner. Thus, if the true sun had passed the observer's anti-meridian by 5 hrs. the

			hrs.	mins.
	L.A.T.	is	05	00
Defined	H.A.	is	17	00
Alternative	H.A.	is	7 hrs. east.	

Example 1. To find longitude from ex-meridian altitude of sun.

Data known or found direct

G.M.T. and G.D. at place	14.00 23–10–36.
True Alt. of sun	26° Bearing approx. N. 158° W.
D.R. Lat.	50° N.
Dec. for sun for G.M.T.	11° 30′ S.
E. for sun for G.M.T.	12 hr. 15 mins. 38 secs.

D.R. Long. Somewhere west of Greenwich in the Atlantic.

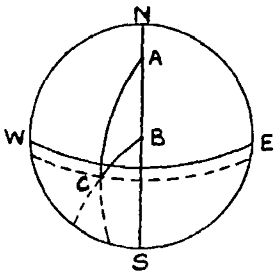


Fig. 10

From above

(*a*) Co-Alt. = 64°.
(*b*) Polar Distance = 101° 30′.
(*c*) Co-Lat. = 40° N.
H.A. = Angle A.

Nat. Hav. θ = Nat. Hav. a − Nat. Hav. $(b \sim c)$
Log Hav. A = Log Cosec b + Log Cosec c + Log Hav. θ

	101° 30′	180°
	40	101 30′
$(b \sim c)$	61° 30′	78° 30′. Supplement of b because b over 90°.

Nat. Hav. θ = Nat. Hav. 64° − Nat. Hav. 61° 30′

Nat. Hav. 64° =	0·28081
Nat. Hav. 61° 30′ =	0·26142
Nat. Hav. θ =	0·01939
Log Hav. θ =	8·28756
Log Cosec 78° 30′ =	10.00881
Log Cosec 40° =	10.19193
Log Hav. A =	8·48830

∴ A = 1 hr. 20 mins. 50 secs.

H.A. = 1 hr. 20 mins. 50 secs. west. West because sun's azimuth is west.

H.A. by calculation 1 hr. 20 mins. 50 secs. west. Converted into the 24 hrs. notation this becomes 1 hr. 20 mins. 50 secs.

	hrs.	mins.	secs.	
G.M.T.	14	00	00	
E.	12	15	38	
	26	15	38	
H.A.	1	20	50	
	24	54	48	
	24			
Longitude		54	48	west.

= 13° 42′ W. West because result is between 0 hrs. and 12 hrs.

Example 2. Find longitude from sun given—

Calculated H.A. 5 hrs. east.
G.M.T. 22 hrs. 00 mins. 7–4–37.
E 11 hrs. 57 mins. 52 secs.

	hrs.	mins.	secs.		hrs.	mins.	secs.	
G.M.T.	22	00	00		24	00	00	
E.	11	57	52		14	57	52	
	33	57	52	Longitude	9	02	08	east.
H.A.	19	00	00					
	14	57	52					

= 135° 32′ E. East because result from formula is between 12 hrs. and 24 hrs.

Note. When the sun is at its meridian altitude its H.A. is 0 hrs.

	hrs.	mins.	secs.		hrs.	mins.	secs.	
G.M.T.	08	44	22		24	00	00	
E.	12	15	38		21	00	00	
	21	00	00	Longitude	3	00	00	east.
H.A.	00	00	00					
	21	00	00					

= 45° E.

To Find Longitude from Ex-Meridian Altitude of Moon—Star—Planet

G.M.T. $\sim$ L.M.T. = Longitude.

Data needed

1. G.M.T. and G.D. at place.
2. True Altitude of body.
3. D.R. Latitude.
4. Declination of body for G.M.T. at place, taking care to obtain correct declination for moon and Venus.
5. Polar Distance.
6. *R* for sun for G.M.T. at place.
7. R.A. of body for G.M.T. at place, taking care to obtain correct R.A. for moon and Venus.
8. H.A. of body.

Working

1. Obtain H.A. as for sun.
2. L.M.T. = H.A. + R.A. − *R*.
3. G.M.T. $\sim$ L.M.T. = Longitude.

The difficulty is to know whether the H.A. is as found from the formula or whether it is 24 hrs.—as found. A method to discover this is to write the equation L.M.T. $\sim$ H.A. = R.A. $\sim$ *R*. The approximate D.R. Long. will be known and so by applying this to known G.M.T., the approximate L.M.T. will be known. In the equation each side must come to the same value. If the wrong H.A. has been taken, the sides will not balance, in which case take the H.A. as found by formula from 24 hrs. and proceed to find L.M.T.

Example. To find longitude from Ex-Meridian sight of moon.

G.M.T. 22.09.

H.A. of moon as found by formula 1 hr. 54 mins. 26 secs.

R.A. of moon 4 hr. 32 mins. 9 secs.

R of sun 4 hrs. 30 mins. 28 secs.

D.R. Long. 0° 30′ W.

As the D.R. Long. is 0° 30′ W., the L.M.T. must therefore be nearly the same as G.M.T.

$$\text{L.M.T.} \sim \text{H.A.} = \text{R.A.} \sim R$$
$$22 \sim 2 \Rightarrow 4\tfrac{1}{2} \sim 4\tfrac{1}{2}$$

The sides do not balance and therefore the H.A. must be 24 hrs. − 1 hr. 54 mins. 26 secs. = 22 hrs. 5 mins. 34 secs.

			H.A.	22	05	34
			R.A.	04	32	09
Longitude = G.M.T. ~ L.M.T.				26	37	43
			R	04	30	28
			L.M.T.	22	07	15
G.M.T.	22	09	00			
L.M.T.	22	07	15			
Longitude		1	45 W. = 26′ 15″ W.			

West, because Greenwich Time Best.

Note. When a star, a planet or the moon is at its meridian altitude, the Hour Angle of the body being observed is 0 hrs. or 24 hrs.

To Find the Approximate Time of Meridian Passage of a Star

Self Stationary, Star above the Pole

Data needed

1. Greenwich Date.
2. D.R. Longitude.
3. Approx. value of R for sun for G.D.
4. Star's R.A.

L.M.T. of passage = Star's R.A. − R. If R.A. less than R, add 24 hrs. to R.A.

Star below Pole

L.M.T. of passage = Star's R.A. + 12 hrs. − R.

Knowing D.R. Long. the G.M.T. of passage may be found.

To decide whether the star is above or below the Pole

Certain stars never go below the observer's horizon. They get higher and higher above the horizon until they reach their meridian altitude, when they are said to be *above* the Pole. They then get lower and lower, but still remain above the horizon, and when they reach their lowest point above the horizon they are said to be *below* the Pole. If, therefore, a star's declination is of contrary name to the latitude, a meridian altitude *below* the Pole will not always be possible to observe because the star may not be visible, having gone below the horizon. Stars which do not go below the horizon are not visible in the daytime because of the light from the sun. Those stars whose declination is greater than and of the same name as the co-latitude will not go below the observer's horizon.

Example. To find approx. time of Mer. Pass. of a star.

1. G.D. 30–1–37.
2. D.R. Long. 80° E.

	hrs.	mins.	secs.
3. Approx. value of *R*	08	38	32.
4. R.A. Sirius	06	42	25.

Required G.M.T. of Mer. Passage.

	hrs.	mins.	secs.
R.A.	30	42	25
R	08	38	32
L.M.T. of Pass.	22	03	53
Long. 80° E.	5	20	00
G.M.T. of Pass.	16	43	53

To Find Approximate Time of Meridian Passage of Sun

Sun Self Stationary

G.M.T. of passage = 24 hrs. − E., + West Long. in time, or − East Long. in time.

E is value for sun for G.M.T. noon for G.D. as found from the *Nautical Almanac.*

Knowing G.M.T. of passage, this may be turned into Zone or other forms of time.

To Find Approximate Time of Meridian Passage of Moon and Planets

Self Stationary

Look up G.M.T. of passage in *Nautical Almanac* for Greenwich Meridian and apply correction for longitude in time as instructed in the explanations given in the *Nautical Almanac.* Result is L.M.T. of passage. Unless the daily difference is considerable, the time given for planets may be treated as L.M.T. of passage.

To Find Approximate Time of Meridian Passage of any Body. Self on Move

Data Needed

1. D.R. Latitude.
2. D.R. Longitude.
3. Track Angle.
4. Ground speed in nautical miles per hour.
5. G.M.T. and G.D. at D.R. position.

Working

1. Find G.M.T. of passage for D.R. position.
2. By means of Traverse Tables in *Norie's* find, for track angle, the Departure for distance you will go in nautical miles in the time interval between G.M.T. at D.R. position and G.M.T. of Mer. Pass. at this D.R. position.
3. Turn this Departure so found into *d*/Long. by means of the tables following the Traverse Tables.
4. Turn *d*/Long. into time.
5. If course is towards east, subtract the time so found from the time of Mer. Pass. at the original D.R. position. If course is towards west, add time to time of Mer. Pass. at the original D.R. position. Result is G.M.T. of passage while self on move.

Note. In turning Dep. into *d*/Long., the latitude to use will be the mean D.R. Lat. at time of observation. No particular accuracy is required.

Examples. To find approx. time of Mer. Pass. of sun. Self stationary.

Date: 30–1–37.
Position Cape Town Lat. 34° S. Long. 18½° E.
Value of *E* for noon 30–1–37 11 hrs. 46 mins. 36 secs.

G.M.T. of passage = 24 hrs. − *E* − East Long.

	hrs.	mins.	secs.
	24	00	00
E	11	46	36
	12	13	24
Long. 18½° E	1	14	00
G.M.T. of Passage	10	59	24
L.M.T. of passage	12	13	24
Zone Time (− 1) ,,	11	59	24
Standard Time (− 2) ,,	12	59	24

To find approx. time of Mer. Pass. of sun. Self on move.

Given—

1. D.R. Lat. . . . . . 34° S.
2. D.R. Long. . . . . . 18½° E.
3. Track Angle . . . . . N. 30° E.
4. Ground Speed . . . . 125 m.p.h.
5. *E* for G.M.T. noon for G.D. . 11 hrs. 46 mins. 36 secs.
6. G.M.T. of D.R. position . . 10 hrs. 20 mins. 30–1–37.

Working

1. G.M.T. of passage at D.R. position 10 hrs. 59 mins. 24 secs.
2.

hrs.	mins.	secs.	
10	59	24	
10	20	00	
	39	24	at 125 m.p.h.

= 82 statute miles = 71 nautical miles
(Ferguson Proportion Calculator.)

Traverse Tables for angle 30° and Distance 71′ give Departure 35·5′. This turned into *d*/Long. by tables following Traverse Tables gives *d*/Long. 42·8′ under Lat. 34°.

d/Long. 42·8′ = 2 mins. 50 secs in time.

	hrs.	mins.	secs.
G.M.T. of passage at original D.R. position	10	59	24
Course towards east		2	50
G.M.T. of Mer. Pass. while under way	10	56	34

To Find Approximate Meridian Altitude of Body

Data needed

1. Date.
2. D.R. Latitude.
3. D.R. Longitude.
4. Declination at G.M.T. for L.M.T. of approx. Mer. Alt. (In case of sun, L.M.T. of Mer. Alt. will be about 12.00 L.M.T. Knowing D.R. Long., convert this L.M.T. to G.M.T. and find the declination for this G.M.T. In case of star or planet—except Venus—take declination as given in the N.A. for date with no further correction for time or longitude position. In case of moon and Venus, find G.M.T. of passage from N.A. for the Greenwich Meridian. Correct this for longitude—see N.A. examples—so getting L.M.T. of passage. Convert this to G.M.T., knowing longitude and find declination for this time. In the Northern Hemisphere take the time given for "Upper" in the case of the moon.

Working

1. Subtract latitude from 90°, giving Co-Lat.
2. If Lat. and Dec. same name, add Dec. to Co-Lat. If more than 90°, subtract from 180°.
3. If Lat. and Dec. contrary names, subtract Dec. from Co-Lat., or vice versa. Result is approx. Meridian Altitude.

Example

To find approx. Mer. Alt. of moon at Lat. 20° N., Long. 90° E.
Local Civil Date 3–4–37

	hrs.	mins.	secs.	
Moon passes Greenwich Meridian	05	27	00	3–4–37
$\frac{90}{360} \times 50$ (*Nautical Almanac*)		12	30	
L.M.T. of passage	05	14	30	3–4–37
Long. 90° E.	6	00	00	
G.M.T. of passage	23	14	30	2–4–37

Moon's declination at 22.00 2–4–37	S. 22° 24·8′
Difference for 2 hrs. 5′ decreasing	3·1
Dec. at 23 hrs. 14½ mins. G.M.T. 2–4–37 or 05 hrs. 14½ mins. L.M.T. 3–4–37	S. 22° 21·7′

	90°
Latitude	20 N.
Co-Latitude	70 N.
Dec.	22 21·7′ S.
Approx. Mer. Alt.	47° 38·3′

To Find Latitude from Meridian Altitude of Body

Data Needed

1. G.M.T. and G.D. of passage.
2. D.R. Longitude.
3. True Altitude.
4. Dec. of body for G.M.T. and G.D. at place.
5. Whether Latitude is north or south.

Working

1. Subtract True Alt. from 90°, so getting Meridian Zenith Distance (Co-Alt.). (M.Z.D.)

2. If Dec. same name as latitude, add Dec. to M.Z.D.

3. If Dec. contrary name to latitude, subtract Dec. from M.Z.D. Result is latitude of place.

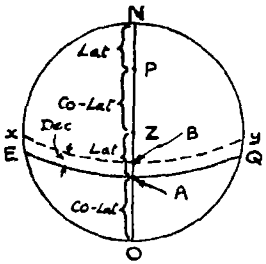


FIG. 11

BO = True Altitude.
ZB = M.Z.D.
Latitude $ZA = ZB + BA$ (Dec.).
NO = Meridian.
P = Pole.
Z = Zenith.
EQ = Equator.
xy = Parallel of declination.

Example. To find Latitude from Meridian Altitude of Body.

The most difficult body to deal with is the moon because it changes its declination rapidly. Of the four planets, Venus, Mars, Jupiter and Saturn, only Venus causes any trouble and requires a correction for the declination given in the *Nautical Almanac* for the day.

Local Civil Date 3–4–37. Early morning.
Moon's Meridian Altitude, True, 47° 38′.
Latitude, Northern Hemisphere.
G.M.T. 23 hrs. 14 mins. 30 secs.
D.R. Long. 90° E.

Long. 90° E. = 6 hrs. 00 mins. G.D. must therefore be 2–4–37.
Dec. for G.M.T. 23 hrs. 14 mins. 30 secs. 2–4–37 = 22° 21·7′ S.

	90°
True Alt.	47 38′
M.Z.D. (Co-Alt.)	42 22
Dec.	22 22 S.
Latitude	20° 00′ N.

To Obtain a Fix from Three Different Observations at Different Times while under Way

Data needed

1. D.R. position for time of middle observation.
2. D.R. Track Angle and ground speed (estimated).
3. Time of observations and hence distance travelled (estimated) between observations.
4. Azimuth of body.
5. Intercept.

Working

See Chart for calculation for intercept and azimuth.

Diagram

This is drawn on a separate piece of paper—called a plotting chart—to the chart being used.

1. Mark any spot on the plotting chart for the mean D.R. position.

2. From this spot draw the azimuth lines having drawn a vertical datum line to represent north and south first.

3. Through the spot, on both sides of it draw the D.R. Track Angle line.

4. Mark off the intercepts on the azimuth lines and draw in the position lines. Let the first and third position lines cut the Track Angle line.

5. From a point where the first position line cuts the Track Angle line, mark off, in the direction of flight on the Track Angle line, the distance travelled between the first and third observation.

6. Draw a line through this point parallel to the first position line so as to cut the third position line intercept.

7. From point where second position line cuts D.R. Track Angle line, mark off, in direction of flight on Track Angle line, the distance travelled between the second and third observations.

8. Draw a line through this point parallel to the second position line so as to cut the third position line intercept.

9. The assumed position for the time of the third observation is taken to be in the centre of the triangle formed by the first and second position lines transferred as above, and the third position line.

10. To measure distances on the plotting chart, make a diagram as illustrated, using any convenient scale for longitude. The angle between the horizontal line and inclined line is that of the latitude. The longitude line is divided up into convenient distances to any scale and lines are drawn perpendicular to the longitude line long enough to cut the horizontal line. When transferring distances, change of latitude or longitude to the actual chart, use the actual printed chart scales, and not the plotting chart diagram scales.

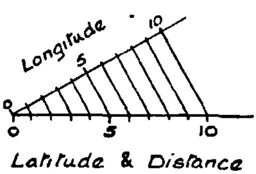


FIG. 12

Corrections to Apply to Observed Altitudes

Marine Type Sextant

Sun	*Moon*	*Planets*	*Stars*
1. Index Error. 2. Total Corrections. *Norie's*, page 194.	1. Index Error. 2. Horizontal Parallax. *Nautical Almanac* for month and day. 3. Total corrections. *Norie's*, page 212.	1. Index Error. 2. Dip and Refraction. 3. Parallax. (Ignore for aeronautics.)	1. Index Error. 2. Dip and Refraction. *Norie's*, page 195.

Note. If the height above sea-level, or above the cloud layer, is more than that provided for in *Norie's Tables*, deduct the square root of the height in feet, calling the square root of the height in feet minutes of arc. Thereafter add 15′ for semi-diameter in respect of both the moon and the sun when using their lower limbs.

The above corrections would probably not be accepted as sufficiently correct for use at sea, but it is suggested that they are sufficiently accurate for use at sea by those who would be learning from this book. They are all that is needed for work in the air.

Norie's Tables describe what has to be done with all corrections other than Index Error. Index Error is a measurement error due to a sextant being out of perfect adjustment. The student will find out all about this when handling the sextant.

Bubble Sextant

Sun	*Moon*	*Planets*	*Stars*
1. Index Error. 2. Parallax and Refraction combined. *Norie's*, page 197.	1. Index Error. 2. Horizontal Parallax. *Nautical Almanac* for month and day. 3. Refraction and Parallax. *Norie's*, page 202.	1. Index Error. 2. Refraction. *Norie's*, page 197. (If observed altitude is over 20°, correction for Index Error only is necessary.)	1. Index Error. 2. Refraction. *Norie's*, page 197. (If observed altitude is over 20°, correction for Index Error only is necessary.)

Note. Norie's Tables show what has to be done with all except Index Error.

The advantage of a bubble sextant is that no correction for Dip is necessary and so it is not necessary to know the observer's height above sea-level or above the cloud horizon. The bubble sextant needs no horizon.

Index Error is the amount the instrument itself reads false as regards the altitude of the body being observed. Certain books and articles on navigation instruct the owner of the bubble sextant to take it to the sea and sit down as near sea-level as possible and then bring the sea horizon down to the horizontal at which point the sextant reading should be zero. If it is off the arc, the Index Error is +, and if on the arc −. The error being the amount on or off.

This method is undoubtedly the best, but it is not always convenient to go to the seaside. It is suggested that a much better practical method is to know your latitude from some reliable map. Using this latitude, work out what the sun's true meridian altitude should be and compare this with what the sextant reads. The difference would be the Index Error. If the sextant reads more than it should, the Index Error is −, and if less than it should read, +.

Bubble Sextant

Observed altitude of moon 3–4–37 was 47° 00′. Index Error + 2′. What is the True Altitude?

Obs. Alt.	47° 00′
I.E. + 2′	02
	47 02
Hor. Parl. 54·7′	
Refr. & Parl. (approx.)	+ 36
True Altitude	47° 38′

TO DRAW A SINGLE POSITION LINE

SUN

Data needed

1. True Altitude of body.
2. D.R. Latitude.
3. D.R. Longitude.
4. G.M.T. at place, and G.D.
5. *E* for G.M.T. at place.
6. Dec. for G.M.T. at place.
7. H.A.

Calculation

1. Bygrave Position Line Slide Rule gives Calculated Altitude and Azimuth. Azimuth is S. if the D.R. Lat. is N. and vice versa.

Azimuth is E. if H.A. is E., and W. if H.A. is W. Subtract calculated altitude from 90° to get Calculated Zenith Distance.

2. Nat. Haversine Formula (see chart) gives the Calculated Zenith Distance.

3. Obtain Azimuth from *Burdwood's Tables*, or A, B, and C Tables in *Norie's* if C.Z.D. has been obtained by Haversine Formula.

4. If Observed Zenith Distance is less than Calculated Zenith Distance, move position line nearer sun and vice versa.

Diagram

1. Mark a spot on the map or chart where you think you are at the time of the observation. This will be marked for the D.R. Lat. and Long.

2. Draw in the azimuth line. This is drawn from the marked spot so that the line represents the azimuth or bearing from the spot to the observed body.

3. Through the D.R. position draw a line at right angles to the azimuth line on both sides of the marked spot.

4. Mark off the intercept distance on the azimuth line from the marked spot, using the scale of the map or chart. The intercept is the difference between the O.Z.D. and C.Z.D. converted into nautical miles by subtracting one angle from the other, minutes of arc being nautical miles.

5. Draw a line through the new position which has been marked on the azimuth line so that the line to be drawn is parallel to the original line drawn through the D.R. position. This new line is the actual position line for G.M.T. of observation and is part of the circumference of a circle called a Circle of Position. You are somewhere on this position line at the G.M.T. of observation. You do not, however, know whereabouts on this position line you are situated.

Note. In this, as in many other examples, the time has been referred to as "G.M.T. at place." If it was 10.00 hrs. G.M.T. in London it would be 10.00 hrs. G.M.T. in New York, or anywhere else in the world, although the local time would be quite different. It is to emphasize the fact that it is the G.M.T. which is required that the words "G.M.T. at place" have been used. To many this wording will be superfluous, but it is considered necessary nevertheless.

To Draw a Single Position Line

STARS, MOON, PLANETS

Data needed

1. True Altitude of body.
2. D.R. Latitude.

3. D.R. Longitude.
4. G.M.T. at place and G.D.
5. *E* of sun for G.M.T. at place.
6. *R* of sun ditto.
7. R.A. of body ditto.
8. Dec. of body ditto.
9. H.A. of body ditto.

Calculations

Same as for sun, but H.A. obtained in a different way. (See Chart.)

Diagram

Same as for sun.

Dec. and R.A. of Moon and Planets

As the declination and right ascension changes rather rapidly, the amount for actual G.M.T. at place must be found. The way in which this is done is explained in the *Nautical Almanac* at the end of the Proportional Parts Tables.

Use of Single Position Line

1. If it is desired to find an island, work out in advance for some known body such as the sun what its azimuth will be for the E.T.A. (estimated time of arrival). Draw in the azimuth, and the position line at right angles to the azimuth through the place. Set a course which will take you to one known side of the place. When the position line has been reached, set a course to make good the position line so as to pass over the place.

2. A body in the same azimuth as the track, or nearly so, will permit of a position line being at right angles to the track. This will allow the ground speed to be checked.

3. A body at right angles to the track, or nearly so, will bring the position line parallel to the track line. This will allow the track to be checked.

4. The moon is often visible in the daytime, and, if it happens to be visible, use of it may be made in conjunction with the sun to get a fix, especially when the angle between them is about 90°.

To Find a Place by Means of a Single Position Line

Data available

Date: 30–1–37.

Place to be found: Birmingham, Lat. 52° 31′ N. Long. 1° 47′ W.

Place of departure: Heston Airport, Lat. 51° 29′ N. Long. 0° 24′ W.

Time of proposed arrival at Birmingham: 14.00 G.M.T.

Distance: Heston to Birmingham, 98 statute miles.

Estimated ground speed on day: 115 m.p.h.

True Rhumb Line course: 320°.

Magnetic Variation: 12° W.

Nautical Almanac and *Norie's Tables* also available.

Working

30–1–37	G.M.T.	14	00	00	Dec. 17° 40′ S.
	E	11	46	35	
		25	46	35	
		24			
	G.A.T.	1	46	35 p.m.	
	Long. 1° 47′ W.		6	08	
	L.A.T.	1	40	27 p.m.	∴ H.A. = 1 hr. 40 min. 27 sec. W.

A, B, & C Tables for Azimuth

Table A.	H.A. 1.40	Lat. 52° 30′	2·8′ S.
,, B.			·76 S.
			3·56′ S.

,, C. Azimuth S. 24½° W. at 1.40 p.m. L.A.T.

Altitude of Sun

By calculation, the altitude of the sun at Birmingham at 14.00 G.M.T. will be 16° 30′.

Procedure

Arrange to make good a track east of Birmingham. When a sextant reading shows the true altitude to be 16° 30′, set 294° + 12° on the compass = 306° and the aircraft should then be flying on the position line towards Birmingham, apart from any correction needed for drift. Arrange to leave Heston at 13.09 G.M.T. as the trip should take 51 minutes (Ferguson Proportion Calculator). Provided the course is altered when the sun's altitude is 16° 30′, it does not much matter whether the ground speed is accurate or not, because at this instant you will be on a circle of position, the circumference of which passes through Birmingham. As the sun moves in azimuth roughly 1° in 4 mins. of time, and as a few degrees in azimuth will not make much difference to the direction of the

position line, a time error of 10 minutes or so will make very little difference to the course needed to pass within sight of the objective. Naturally it may not be possible to observe the sun when its altitude is exactly 16° 30′, and so the tables in *Norie's* for "Change of Altitude in one minute of Time Azimuth" are brought into use in order to know the time interval to when the sun will be 16° 30′. Suppose a sight has been obtained and the true altitude is found to be 16° 40′ (it being remembered that as the H.A. is west, the sun is on its way towards setting). A quick glance in the tables against the approximate latitude 52° and approximate azimuth 24° will show that the change in altitude is about 3½′ per minute of time. The course must therefore be changed about 2½ mins. after the 16° 40′ sight.

It will depend on what distance away to the east of Birmingham you have aimed to hit the position line as to how long you will fly on the altered course. A distance of 10 miles would be a suitable amount, and in this distance the effect of drift could be ignored unless the wind was very strong. If the wind was strong, the visibility would usually be good.

In theory the foregoing sounds very simple, and so in theory it is. The practical difficulty lies in holding the sextant steady in order to get an accurate reading and in being able to obtain an uninterrupted view of the observed body. In calm air the system has a lot to recommend it, provided the navigator can get a view of the sky, as it is extremely simple and is almost foolproof. The Bygrave Slide Rule is ideal for working out the altitude and azimuth, taking about three minutes in which to do so. If the system is used at night, select a star which has south declination if you are in the northern hemisphere, and vice versa so as to get an object which will rise and set. It will not do to take a circumpolar star. (See page 46.)

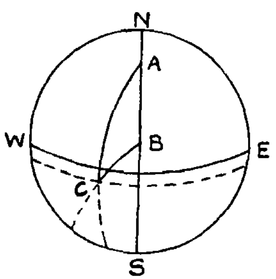


FIG. 13

Haversine Formula (two sides and included angle)

A = 1 hr. 40 mins. 27 secs. (Hour Angle).
b = 107° 40′ or 72° 20′ (Polar Distance).
c = 37° 29′ (Co-Latitude).

$$
\begin{aligned}
\text{Log Hav. A} &= 8{\cdot}67451 \\
\text{Log Sin } b &= 9{\cdot}97902 \\
\text{Log Sin } c &= 9{\cdot}78428 \\
\text{Log Hav. } \theta &= \overline{8{\cdot}43781} \\
\text{Nat. Nav. } \theta &= 0{\cdot}02741 \\
\text{Nat. Hav. } (b \sim c) &= \;\;{\cdot}33049 \; (107^\circ\, 40' \sim 37^\circ\, 29') \\
\text{Nat. Hav. } a &= \overline{0{\cdot}35790}
\end{aligned}
$$

$$
\begin{array}{lll}
 & 90^\circ & 00' \\
a & 73 & 29\tfrac{1}{2} \\
\hline
 & 16^\circ & 30\tfrac{1}{2}'
\end{array}
\qquad \therefore a = 73^\circ\, 29\tfrac{1}{2}'
$$

Altitude = 16° 30½′.

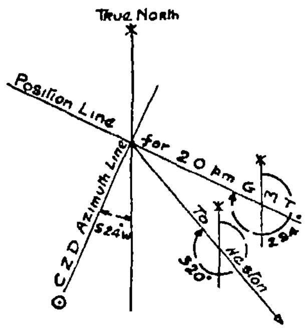


FIG. 14

To Find Latitude from Pole Star

Data needed.

1. D.R. Longitude.
2. G.M.T. at place and G.D.
3. R for Sun for G.M.T. at place and G.D.
4. True Altitude of Polaris.

Working

1. G.M.T.	......	
Long.	......	
L.M.T.	......	
R	......	
R.A.M. or L.S.T.	......	Deduct 24 hrs. if more than 24 hrs.

2. Turn to Pole Star tables in *Nautical Almanac* and tabulate corrections as found in Tables I, II, and III.

3. Apply corrections to True Altitude. Result is Latitude of place. (Ignore table corrections II and III for use in the air.)

Example

D.R. Long. 45° W.
G.M.T. 22.30 30–1–37.
Observed Alt. Polaris 48° 15′. Bubble Sextant. I.E. + 2′.

G.M.T.	22	30	00	30–1–37
Long. 45° W.	3	00	00	
L.M.T.	19	30	00	
R	08	38	56·3	
	28	08	56·3	
	24			
R.A.M. or L.S.T.	04	08	56·3	

Table I.	−	49·4′
,, II.	+	0·2
,, III.	+	·1
	−	49·1′

True Alt.	48°	17′
Table Corr. −		49
Lat. of place	47°	28′

Note. So long as the D.R. Longitude is known within 1° or so, the resultant error in Lat. is very slight.

To Find what Stars and Planets are Due to be at Their Meridian Altitude between Limits of Time

Data needed

1. G.M.T. and G.D. at place.
2. D.R. Longitude.
3. Value of *R* for Sun for G.M.T. and G.D.

Working

1. Find L.M.T. by applying the correction for Longitude.

2. Add *R* to L.M.T. If more than 24 hrs. deduct 24 hrs. Result is R.A.M.

3. Set out work in two columns, each column headed by the respective G.M.T. limits of time. This will give two R.A.M.'s. Stars having a R.A. between these two R.A.M.'s will be at their meridian altitude between these limits of time.

4. Look in *Nautical Almanac* for given month to find what stars have a R.A. between the R.A.M. limits found above.

Example What stars or planets up to 2nd Magnitude are due to be at their meridian altitude between 18.00 and 19.00 G.M.T. on 30–1–37? D.R. Long. 5° east.

G.M.T.	18	00	00	19	00	00	30–1–37
Long. 5° east		20	00		20	00	
L.M.T.	18	20	00	19	20	00	
R	08	38	12	08	38	22	
	26	58	12	27	58	22	
	24			24			
R.A.M.	02	58	12	03	58	22	

Star: *α* Persei–Mirfak.
Planets: nil.

To Calculate Ex-Meridian Altitude of Body at any Given Time

Data needed

1. D.R. Latitude.
2. Hour Angle of body at time altitude is needed.
3. Declination of body at time altitude is needed. (If the moon is being used, take correct proportion for Dec.)

Working

1. Calculate C.Z.D. by Haversine Formula. (See Chart.)
2. Subtract C.Z.D. from 90° giving Calculated Altitude.

Note. If the Bygrave Slide Rule is used, it gives the altitude direct.

The difficulty is to find the Hour Angle.

To Convert Altitude of Body at a Given Time to what it would have been at any Other Time

Data needed

1. D.R. Latitude.
2. Azimuth.
3. Time of observation.

Working

See explanation in *Norie's Tables* under heading "Change of Altitude in one minute of Time, Azimuth," page xxxii and 221.

Example

D.R. Lat. 51° 30′ N.
Azimuth N. 140° E.
Time 18 hrs. 36 mins. any variety.
True Altitude of body at 18.36 = 26° 15′.
What would the True Altitude be at 18.30, same time units?

Norie's, page 221.	Lat. 51°	Azimuth	S. 40° E.	6·0′
	Lat. 52°	,,	,,	5·9
	Lat. 51° 30′			5·95
				6
				35·70′

18.36 Alt.	26°	15′
Correction		35·7
18.30 Alt.	25°	39·3′

Note. The altitude is less because the body bears to the east of the observer's meridian and so has not reached its meridian altitude. All bodies appear to rise in the east and set in the west. They reach the highest point they will get in the sky when they are opposite the observer's meridian.

To Compute the Hour Angle of the Sun

Data needed

1. G.M.T. and G.D. at place.
2. D.R. Longitude.
3. *E* for sun for G.M.T. and G.D. at place.

Working

See chart.

To Compute the Hour Angle of Moon, Planets, Stars

Data needed

1. G.M.T. and G.D. at place.
2. D.R. Longitude.
3. *R* for sun for G.M.T. and G.D. at place.
4. R.A. for body for G.M.T. at place. (In case of moon, apply correction for longitude as directed in explanation in *Nautical Almanac*.)

Working

See chart.

Note. The Hour Angle so computed relies on the D.R. Longitude being fairly accurate.

To Calculate Latitude from Ex-Meridian Observation

Data needed

1. Hour Angle.
2. Declination.
3. True Altitude of body.

It is of no use to calculate the H.A. from D.R. Latitude for this. The H.A. must be obtained from D.R. Longitude or by calculation from the measured azimuth and altitude of the body. If the H.A. is calculated using a D.R. Lat., the resultant calculation for actual latitude will merely give the same D.R. Lat.

Working

1. (1) Find Polar Distance from declination.
   (2) Find Co-Alt. from true altitude.
   (3) Solve problem by dividing the triangle into two right-angled triangles. Two sides and angle opposite one of them.

2. This problem may also be solved by means of the modified Haversine Formula, provided that the time of observation is not too much before or too much after the object has reached its meridian altitude. (See below.)

To Calculate Latitude from Ex-Meridian Observation by Modified Haversine Formula

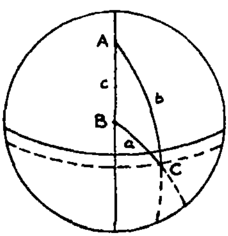


Fig. 15

A = Pole and H.A.

B = Zenith.

AB = Co-Lat. obtained from D.R. Lat.

BC = Zenith Distance. $90° -$ True Alt.

AC = Polar Distance. $90° \pm$ Dec.

If BC was on the meridian instead of being off to the side, then $BC \pm$ Dec. is the latitude. When the H.A. is small, i.e. when the sight has been taken shortly before or after the meridian passage, then BC is nearly the same length as what it would have been had it been on the meridian. Therefore,

BC is, to all intents and purposes, equal to the Meridian Zenith Distance, and so $b \sim c$ = Mer. Zenith Distance (nearly).

1. Log Hav. θ = Log. Hav. A + Log Sin b + Log Sin c.
2. Nat. Hav. a = Nat. Hav. θ + Nat. Hav. $(b \sim c)$
   or Nat. Hav. $(b \sim c)$ = Nat. Hav. a − Nat. Hav. θ.

$\therefore$ Nat. Hav. Mer. Zenith Distance = Nat. Hav. a − Nat. Hav. θ.

		°	′	″
$\therefore$ M.Z.D.	=	-	-	-
Dec.	=	-	-	-
Actual Lat.		-	-	- (See Lat. from Mer. Alt. of body.)

Note. See *Norie's Tables*, xviii, for change of longitude for change of latitude.

To Calculate an Hour Angle for Finding Longitude or Other Purposes than for Finding Latitude

Data needed

1. True Altitude of body.
2. D.R. Latitude.
3. Declination.
4. Polar Distance.

Working

Solve by Haversine Formula (three sides known).

Sides known are—

Co-Altitude or Zenith Distance.
Co-Latitude.
Polar Distance.

Note. The H.A. so calculated must not be used to find latitude from an Ex-Meridian observation. Further, it is the angle E. or W. of the observer's meridian.

To Calculate an Hour Angle knowing neither Latitude nor Longitude

Data needed

1. True Altitude of body.
2. Azimuth of body as measured by compass corrected to True.
3. Chart showing Magnetic Variation of vicinity.
4. Deviation Card for compass.
5. Declination for G.M.T. and G.D.

Working

1. Convert altitude into Co-Alt. or Zenith Distance.
2. Obtain Polar Distance from Dec.
3. Correct compass bearing for Deviation and Magnetic Variation so obtaining the true bearing. Measure the angle from the Pole of the hemisphere in which you are situated from 0° to 180° through east or west as the case may be. For example, if the true bearing is less than 180° and the Northern Hemisphere Pole is being used, call the azimuth N. . . . E. If more than 180°, subtract it from 360° and call the azimuth N. . . . W.
4. $$\frac{\text{Sin H.A.}}{\text{Sin Zenith Distance}} = \frac{\text{Sin Azimuth}}{\text{Sin Polar Distance}}$$

   If any of the angles are more than 90°, subtract the angle from 180°.

   The formula twisted round becomes—

   $$\text{Sin H.A.} = \text{Sin Azimuth} \times \text{Sin Zenith Distance} \times \text{Cosec Polar Distance.}$$

5. The H.A. so found will be less than 90°, but an Hour Angle may be anything up to 360° or 24 hrs. Unless the approximate L.M.T. is known it will be difficult to say what the actual H.A. should be. To know the L.M.T. the approximate longitude must be known. If the L.M.T. is known approximately, then the sides of the formula for bodies other than the sun

$$\text{L.M.T.} \sim \text{H.A.} = \text{R.A.} \sim R$$

must balance. If the sides do not balance, the wrong value has been taken for the Hour Angle. (See page 24.)

In the case of the sun, the azimuth will give an idea of what the Hour Angle should be, because, if the azimuth is N. 158° W., the H.A. must be less than 12 hrs. because the sun has passed the meridian on its way towards setting. (See pages 20–21.)

Hour Angles and Longitude at Meridian Altitudes

Sun

When the sun is at its Meridian Altitude the Hour Angle is 0 hrs. and L.A.T. is 12 hrs.

G.A.T. ~ L.A.T. = Longitude

	hrs.	mins.	
∴ If G.A.T. was	14	00	
and L.A.T. was	12	00	
The Longitude is	02	00	west.

= 30° W. west because Greenwich Time Best.

Stars

When a star is at its Meridian Altitude the Hour Angle of the star is 24 hrs. or 0 hrs.

$$\text{G.M.T.} \sim \text{L.M.T.} = \text{Longitude.}$$

Example. Sirius is at its Meridian Altitude. What is the longitude given the following?

G.M.T. 22 hrs. 25 mins. 25 secs.
D.R. Lat. Any latitude.
D.R. Long. 5° W.
R for sun 08 hrs. 38 mins. 55 secs.
R.A. for Sirius 06 hrs. 42 mins. 25 secs.

$$\text{G.M.T.} \sim \text{L.M.T.} = \text{Longitude. And L.M.T.} = \text{H.A.} + \text{R.A.} - R$$

$$\text{L.M.T.} \sim \text{H.A.} = \text{R.A.} \sim R$$

$$22 - 0 = 7 - 9$$

which does not balance. The H.A. must therefore be 24 hrs.

	hrs.	mins.	secs.
H.A.	24	00	00
R.A.	06	42	25
	30	42	25
R	08	38	55
Actual L.M.T.	22	03	30

	hrs.	mins.	secs.
G.M.T.	22	25	25
L.M.T.	22	03	30
		21	55

Long. 5° 28′ 45″ W. west because Greenwich Time Best.

WHAT STAR IS IT?

Data needed

1. True Altitude of body.
2. D.R. Latitude.
3. G.M.T. and G.D. at place.
4. D.R. Longitude.
5. *R* for sun for G.M.T. and G.T. at place.
6. Azimuth. North via east to south, or north via west to south.

Working

1.

	hrs.	mins.	secs.
G.M.T.	–	–	–
Longitude	–	–	–
L.M.T.	–	–	–
R +	–	–	–
R.A.M.	–	–	–

2. Knowing latitude, altitude and azimuth, find from a little book called Harvey's *What Star is it?* the H.A. and Dec.

3. See rules in *What Star is it?* as to whether the H.A. is to be added or subtracted in conjunction with the R.A.M. to give the star's R.A. Also see rules for finding the declination.

4. Turn up the *Nautical Almanac* for the month and find what star has the R.A. and Dec. as found.

Note. Test this by observing some star known to you. It will be found that the R.A. comes fairly near, but the Dec. may be several degrees out.

To Find what Stars will be Visible during Certain Periods of the Year

1. No star can get below the observer's horizon if its polar distance is less than the latitude of the observer's position. Such stars are called "circumpolar" stars.

2. Any star whose polar distance is exactly the same as the co-latitude will pass directly overhead at some time during the day or night.

3. No star can be visible if its polar distance exceeds 90° + Co-Latitude of the observer's position.

4. As a star's declination is more or less the same all the year round, and as polar distance is 90° ± Declination, the polar distance of stars will remain the same in the same hemisphere all the year round. Anyone who has looked at the stars in the winter will have noted that they can see stars which they do not see in the summer, but in any particular latitude the same stars are there, although they cannot be seen, as they rise and set at a different time and the daylight prevents them from being seen.

5. Having decided that a star should be visible in so far as its polar distance is concerned, find the approximate time of its meridian passage, and this will give an idea as to whether it will be seen or whether there will be too much daylight.

TIME

It is necessary to understand something about time in dealing with navigation. The sun is our timekeeper. It rises in the east and sets in the west in both hemispheres.

There are numerous kinds of time but those it is necessary to understand are given below—

Greenwich Mean Time (*G.M.T.*). The sun does not travel at a uniform rate round the world, and so a sun travelling at a uniform rate is taken called a Mean Sun. Actually the world travels round the sun, but this is immaterial. When the Mean Sun passes the meridian of Greenwich, the time is 12.00 (Mean Noon). It was 00.00 hrs. when the Mean Sun passed the anti-meridian of Greenwich. G.M.T. is therefore the time which has elapsed since the Mean Sun passed the anti-meridian of Greenwich.

Local Mean Time (L.M.T.). Obviously if the Mean Sun passes the Greenwich meridian at 12.00 hrs. G.M.T. it could not at the same instant be opposite the meridian of Plymouth, the longitude of which is roughly 4° W., and although the clocks in Plymouth would record 12.00 G.M.T. when the Mean Sun was passing the meridian of Greenwich, the Mean Sun would not at that time have reached the meridian of Plymouth. The Local Mean Time at Plymouth at this instant would be 12.00 hrs. — 16 mins. = 11.44, from which it will be realized that time and longitude have a definite relationship with each other. (See chart for relationship.) L.M.T. is therefore G.M.T. corrected for longitude.

Greenwich Apparent Time (*G.A.T.*). This is the actual or true sun's time in relation to the Greenwich meridian. It is obtained by adding the value of a quantity called *E*, found in the *Nautical Almanac* under "Sun," to the known G.M.T.—G.A.T. will always be somewhere near G.M.T. In adding the value *E* to the G.M.T. it will be found that it does not come anywhere near the known G.M.T. but will differ roughly by 12 hrs. If the total comes to more than 24 hrs., deduct 24 and call the result p.m. (post-meridian). If the total is less than 24 hrs., subtract 12 hrs. and call a.m. (ante-meridian). (Not the same word or meaning as anti-meridian.)

Local Apparent Time (*L.A.T.*). This is the actual or true sun's time in relation to the local meridian. At Local Apparent Noon the true sun is directly opposite the local meridian. To compute L.A.T. it is necessary to know G.M.T., the value of *E* for G.M.T., and the longitude.

L.A.T. = G.A.T. − west longitude in time, or + east longitude in time.

L.A.T. will be somewhere near L.M.T. in amount. If it is not, apply the same rules as for G.A.T.

Standard Time. This is a form of Mean Sun's time kept for civil use on land in various parts of the world. In England, the Standard Time is the same as G.M.T. (except when Summer Time is in force), but in Egypt the Standard Time differs from G.M.T. by 2 hrs. The *Nautical Almanac* tabulates the Standard Time difference to G.M.T. for various parts of the world. On inspection it will be noticed that the times have a − or a + sign against them. If the sign is −, the Standard Time shown by the clocks of the country at any particular time of day must be subtracted by the amount of the difference shown in the *Nautical Almanac* in order to arrive at G.M.T. Thus, if the time by the clocks in Cairo is 2.0 p.m., the G.M.T. is 12.00 hrs.

Zone Time (*Z.T.*). This is a form of Standard Time used chiefly at sea. The diagram illustrating Zone Time should be studied carefully. To find the Zone Number, divide the longitude of a place to the nearest degree by 15 and write the balance over as a fraction. The Zone Number is the number of times 15 goes into the longitude, but if the fraction is more than 7/15ths, add one to the Zone Number. For example, what is the Zone Number for longitude 100° east? Zone No. − 7. 15 goes into 100 six times and 10 over.

Greenwich Date (*G.D.*). As the time used in different parts of the world is not G.M.T., it follows that the day of the month will also differ in certain instances. The diagram illustrating Zone Time will indicate what is happening. In dealing with astronomical navigation, all times have to be converted to G.M.T. In dealing with time of all kinds remember, Longitude west, Greenwich Time Best. Longitude east, Greenwich Time Least.

Sidereal Time. Considered by most navigators to be all-important It does not appear in this book other than in the form of R.A.M

Zone Times

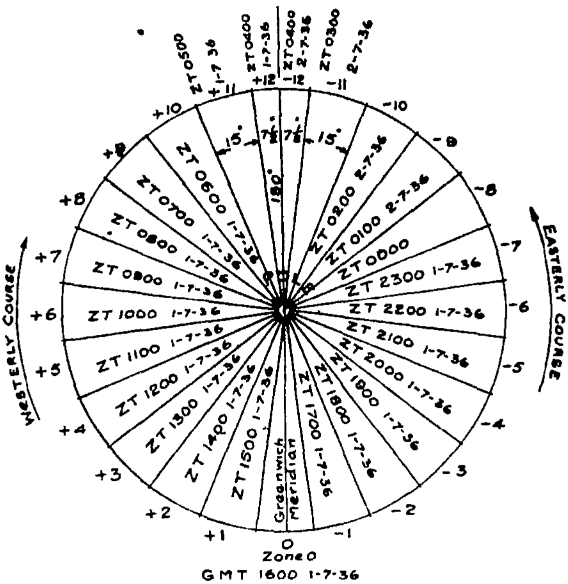


Fig. 16

LONGITUDE WEST—GREENWICH TIME BEST
LONGITUDE EAST—GREENWICH TIME LEAST

Zone Time on no-date meridian. (180°.)

When going from one side to the other in Zone + 12, — 12, the time is the same but the date changes. When crossing the line on a westerly course, the date must be advanced one day. When crossing on an easterly course, the date must be put back one day.

Compass Bearings of Bodies or Objects

A compass needle, north end, points to magnetic north when unaffected by what is called Deviation. Unless there is any local magnetic attraction pulling the compass needle sideways, there will be no deviation. You cannot tell by looking at the compass alone if it has any deviation. If deviation is known to exist, the amount of the deviation must be added to the compass bearing if the deviation is easterly, and subtracted if westerly in order to get the correct magnetic direction. If deviation is present, it will vary according to the position of the disturbing influence in relation to the north end of the compass needle. In dealing with the sun and other

bodies, the true bearing and not the magnetic bearing is needed. The angular difference between true north and magnetic north is called "Magnetic Variation." This may be east or west. Most maps and charts show the amount of the local magnetic variation which may need a correction as it does not remain the same. If the magnetic variation is east it must be added to the magnetic bearing in order to get the true bearing. If it is west it must be subtracted.

If the true bearing is known, in order to convert this into a compass bearing, easterly deviation and/or magnetic variation must be subtracted. Westerly deviation and/or magnetic variation must be added.

The foregoing remarks refer to bearings taken on the 360° method as opposed to the nautical method. In this latter method a true bearing of 220° would be termed S. 40° W.

An azimuth is another term for a bearing.

Examples

It is desired to go from Heston airport to Plymouth aerodrome flying on a Great Circle course, leaving Heston at 8.0 p.m. G.M.T. on 30–1–37 at an average estimated ground speed of 115 m.p.h.

1. What is the distance?
2. What is the outward initial G.C. Course, and return ditto?
3. What is the latitude of the Vertex?
4. What is the longitude of the Vertex?
5. Would the star Sirius reach its meridian altitude while on the way? If not, give the G.M.T. of its meridian altitude.
6. What is the Rhumb Line course and distance?

First find the latitude and longitude of Heston and Plymouth aerodromes. These particulars may be obtained from *The Air Pilot*, Vol. I.

	Lat.	Long.
Heston	51° 29′ N.	0° 24′ W.
Plymouth	50° 25′ N.	4° 07′ W.

	90° 00′	90° 00′	
	51 29	50 25	
Co-Lats.	38° 31′	39° 35′	*D*/Long. 3° 43′.

Haversine Formula
(two sides and included angle).

Required Side CB.

1. $\text{Log Hav. } \theta = \text{Log Hav. } A + \text{Log Sin } b + \text{Log Sin } c$
   $\text{Nat. Hav. } a = \text{Nat. Hav. } \theta + \text{Nat. Hav. } (b \sim c)$

$$\begin{aligned}
\text{Log Hav. } 3^\circ\, 43' &= 7{\cdot}02185\\
\text{Log Sin } 39^\circ\, 35' &= 9{\cdot}80428\\
\text{Log Sin } 38^\circ\, 31' &= \underline{9{\cdot}79431}\\
\text{Log Hav. } \theta &= 6{\cdot}62044\\
\\
\text{Nat. Hav. } \theta &= 0{\cdot}00042\\
b \sim c = 1^\circ\, 04' \qquad \text{Nat. Hav. } 1^\circ\, 04' &= \underline{0{\cdot}00009}\\
\text{Nat. Hav. } a &= 0{\cdot}00051
\end{aligned}$$

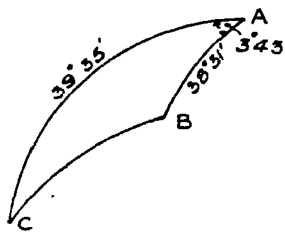


FIG. 17

$\therefore a = 2^\circ\, 35\tfrac{1}{2}' = 155\tfrac{1}{2}'$

	155·5
10%	15·55
5%	7·77
Statute miles	178·82

2. $\dfrac{\text{Sin } B}{\text{Sin } b} = \dfrac{\text{Sin } A}{\text{Sin } a}$

$\therefore$ Sin B = Sin A Sin b Cosec a

$$\begin{aligned}
\text{Log Sin } 3^\circ\, 43' &= 8{\cdot}811726\\
\text{Log Sin } 39^\circ\, 35' &= 9{\cdot}804276\\
\text{Log Cosec } 2^\circ\, 35\tfrac{1}{2}' &= \underline{11{\cdot}344692}\\
&\ 9{\cdot}960694
\end{aligned}$$

$\therefore$ B by calculation = 65° 59½′. Say 66°.

$\dfrac{\text{Sin } C}{\text{Sin } c} = \dfrac{\text{Sin } A}{\text{Sin } a}$

$\therefore$ Sin C = Sin A Sin c Cosec a

$$\begin{aligned}
\text{Log Sin } 3^\circ\, 43' &= 8{\cdot}811726\\
\text{Log Sin } 38^\circ\, 31' &= 9{\cdot}794308\\
\text{Log Cosec } 2^\circ\, 35\tfrac{1}{2}' &= \underline{11{\cdot}344692}\\
&\ 9{\cdot}950726
\end{aligned}$$

$\therefore$ C by calculation = 63° 13′.

Initial Great Circle courses. B to C = S. 66° W. = 246°.
C to B = N. 63° 13′ E. = 63° 13′

3. Cos Lat. Vertex = Cos Lat. B Sin course at B
= Cos Lat. 51° 29′ Sin 66°

Log Cos 51° 29′	=	9·794308
Log Sin 66°	=	9·960730
		9·755038

∴ Lat. of Vertex = 55° 19½′ N.

Cos. Lat. Vertex = Cos Lat. C Sin Course at C
= Cos Lat. 50° 25′ Sin 63° 13′

Log Cos 50° 25′	=	9·804276
Log Sin 63° 13′	=	9·950714
		9·754990

∴ Lat. of Vertex = 55° 19½′ N.

4. Cot Long. from B = Sin Lat. B Tan course at B.

Log Sin 51° 29′	=	9·893444
Log Tan 66°	=	10·351417
		10·244861

∴ Long. from B = 29° 38½′ towards east.

Long of Vertex from B	=	29° 38½′ towards east
Long. of B	=	0 24 W.
Long. of Vertex	=	29° 14½′ E.

So that the highest latitude the G.C. will reach is 55° 19½′ N. and this will be in Long. 29° 14½′ E.

5. Heston Departure 8.0 p.m. G.M.T. 30–1–37. Distance 179 miles at 115 m.p.h. ground speed will take about 1 hr. 34 mins. (Ferguson Proportion Calculator), giving the time limits between 20.00 and 21.34.

	Dep.				Arr.		
G.M.T.	20	00	00		21	34	00
Long. 0° 24′ W.		1	36	Long. 4° 07′ W.		16	28
L.M.T.	19	58	24		21	17	32
R	08	38	32		08	38	47
	28	36	56		29	56	19
	24				24		
R.A.M.	04	36	56		05	56	19

R.A. Sirius is 06 hrs. 42 mins. 25 secs. and so Sirius would not reach its meridian altitude while on the way between Heston and Plymouth.

L.M.T. of passage = Star's R.A. − *R*. If R.A. less than *R*, add 24 hrs. to R.A.

	hrs.	mins.	secs.		hrs.	mins.	secs.
Star's R.A.	06	42	25	or	30	42	25
R for G.M.T. 20.00	08	38	32				
L.M.T. of Mer. Alt.	22	03	53				
Long. 0° 24′ W.		1	36				
G.M.T. of Mer. Alt. at Heston	22	05	29	(Answer).			
Heston Dep. G.M.T.	20	00	00				
Mer. Alt. after Dep.	2	05	29				

In which time, at 115 m.p.h., one will go about 210 nautical miles. The Track Angle is S. 66° W. Turn to Traverse Tables in *Norie's* at 66° and for the distance of 210′ the Departure is found to be 191·8.

Next turn to the tables Departure into *d*/Long., and vice versa. Lat. 50°.

Dep. 19	29·6	*d*/Long.
,, 20	31·1	
Difference	1.5	
Diff. for ·18	·3	

∴ Dep. for 191·8 = 299′ *d*/Long.
= 4° 59′ *d*/Long. going west.
which in time = 19 mins. 56 secs.

Therefore, if you continued your flight beyond Plymouth, Sirius would reach its meridian altitude at 22 hrs. 25 mins. 25 secs. G.M.T.

hrs.	mins.	secs.	
22	05	29	
	19	56	
22	25	25	G.M.T.

and your longitude at this time would be 5° 33′ W.

Original Longitude	00° 24′	W.
d/Long.	4 59	W.
New Longitude	5° 33′	W.

	hrs.	mins.			hrs.	mins.	secs.
G.M.T.	22	25	25				
Long. 5° 33′ W.		22	12				
L.M.T.	22	03	13				
Star's R.A.	06	42	25	or	30	42	25
R for 22.25					08	38	55
				L.M.T.	22	03	30

6.

	Latitude	Meridional Parts	Longitude
Heston	51° 29′ N.	3615·13	0° 24′ W.
Plymouth	50 25 N.	3513·54	4 07 W.
	1° 04′	101·19	3° 43′
	60		60
	64′		223′

$$\text{Tan Course} = \frac{d/\text{Long.}}{d/\text{M.P.}} = \frac{223}{101.19}$$

$$\begin{aligned} \text{Log } 223 &= 2{\cdot}348305 \\ \text{Log } 101{\cdot}19 &= 2{\cdot}005136 \\ &\quad 10{\cdot}343169 \end{aligned}$$

$$= 65° \ 35\tfrac{1}{2}'$$
$$= \text{S. } 65° \ 35\tfrac{1}{2}' \text{ W.}$$

$$\text{Distance} = d/\text{Lat.} \times \text{Sec. Course.}$$

$$\begin{aligned} \text{Log. } 64' &= 1{\cdot}806180 \\ \text{Log. Sec. } 65° \ 35\tfrac{1}{2}' &= 10{\cdot}383801 \\ &\quad 2{\cdot}189981 \end{aligned}$$

$$= 154{\cdot}9 \text{ nautical miles,}$$

which is half a nautical mile less than the Great Circle course. This could not be the actual case because the G.C. course is the shortest distance. The error is due to the distance between Heston and Plymouth being too little to show any marked difference between a G.C. course and Rhumb Line course for the formulae used. The inference to be drawn from the above example is that unless two places are of considerable distance apart, nothing is to be gained by flying or sailing on a G.C. course with its attendant complications in having to change the compass course for change of bearing due to the convergency of the meridians.

Had the Haversine Formula been used instead of the Sine Formula to calculate the initial Great Circle course, it would have been found

that the course angle from Heston to Plymouth comes to 113° 06′. As the start is in the northern hemisphere, the angle becomes N. 113° W., which is S. 67° W., as against S. 66° W. The Sine Formula is not accurate when small angles are being dealt with as in the case of the *d*/Long. and distance in this particular example. The angle of 66° given by the Sine Formula is also not really 66° but 114°. The Sine Formula has been used in this example in order to show up the inaccuracy.

I AM SOMEWHERE IN THE ATLANTIC. WHERE AM I?

If you can find the latitude and longitude, then you know where you are. You have the following available—

1. Bubble sextant.
2. Watch keeping Greenwich time.
3. *Nautical Almanac*.
4. Compass.
5. Chart showing lines of magnetic variation.
6. Sun.

Time. 14.00 G.M.T. 23–10–36.
Observed Alt. of sun 26°. Sextant error nil.
Compass bearing of sun 222°. Deviation nil.
Magnetic Variation for assumed position 20° W., giving sun's true bearing to be 202°. The sun has therefore passed the observer's meridian towards the west because it would have been directly opposite the meridian at local apparent noon. The latitude is north and so the azimuth must be N. 158° W.

The *Nautical Almanac* shows that the sun's declination is about 11° 30′ S. Drawing a diagram the following are known—

AC = Polar Distance = $101\frac{1}{2}$°.
BC = Co-Alt. = 64°.
Angle ABC = 158°.

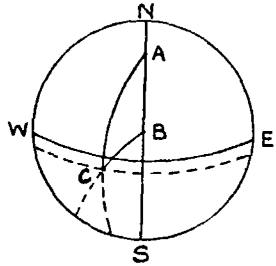


FIG. 18

Two sides and an angle opposite one of them. Required to find the angle CAB, i.e. the Hour Angle, and side AB, i.e. Co-Lat. from which the latitude may be obtained.

$$\frac{\text{Sin } A}{\text{Sin } a} = \frac{\text{Sin } B}{\text{Sin } b}$$

$$\therefore \quad \text{Sin A} = \text{Sin } 158° \text{ Sin } 64° \text{ Cosec } 101° \, 30'$$
$$= \text{Sin } 22° \text{ Sin } 64° \text{ Cosec } 78° \, 30'$$

$$\begin{aligned} \text{Log Sin } 22^\circ &= 9{\cdot}573575 \\ \text{Log Sin } 64^\circ &= 9{\cdot}953660 \\ \text{Log Cosec } 78^\circ\, 30' &= \underline{10{\cdot}008807} \\ & 9{\cdot}536042 \end{aligned}$$

$$A = 20^\circ\, 06'$$

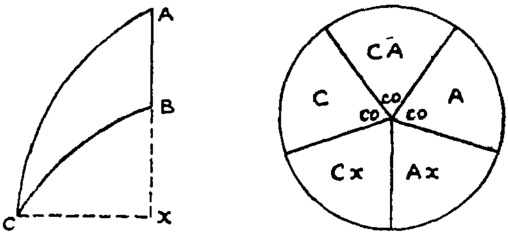


Fig. 19

$$\operatorname{Cos} A = \operatorname{Tan} Ax \operatorname{Cot} CA$$

$$\therefore \quad \operatorname{Tan} Ax = \operatorname{Cos} A \operatorname{Tan} CA$$

$$\begin{aligned} \text{Log Cos } 20^\circ\, 06' &= 9{\cdot}972709 \\ \text{Log Tan } 78^\circ\, 30' &= \underline{10{\cdot}691537} \\ & 10{\cdot}664246 \\ & = 77^\circ\, 46\tfrac{1}{2}' \end{aligned}$$

$= \operatorname{Cos} 20^\circ\, 06' \operatorname{Tan} 78^\circ\, 30'$

$= 77^\circ\, 46\tfrac{1}{2}'$. But really $180^\circ - 77^\circ\, 46\tfrac{1}{2}'$ (See rules for R.A. Triangles, hypotenuse AC being more than 90°.)

$= 102^\circ\, 13\tfrac{1}{2}'$

And $$\frac{\operatorname{Cos} CA}{\operatorname{Cos} Ax} = \frac{\operatorname{Cos} CB}{\operatorname{Cos} Bx}$$

$$\begin{aligned} \text{Log Cos } 64^\circ &= 9{\cdot}641842 \\ \text{Log Cos } 77^\circ\, 46\tfrac{1}{2}' &= 9{\cdot}325826 \\ \text{Log Sec } 78^\circ\, 30' &= \underline{10{\cdot}700345} \\ & 9{\cdot}668013 \end{aligned}$$

$$\therefore \quad \operatorname{Cos} Bx = \operatorname{Cos} CB \operatorname{Cos} Ax \operatorname{Sec} CA$$

$$= \operatorname{Cos} 64^\circ \operatorname{Cos} 77^\circ\, 46\tfrac{1}{2}' \operatorname{Sec} 78^\circ\, 30'$$

$$\therefore \quad Bx = 62^\circ\, 15'.$$

Co-Lat $AB = Ax - Bx = 40^\circ$ nearly.

$\therefore$ Latitude $= 50^\circ$ N.

$$\begin{array}{r} 102^\circ\ 13\tfrac{1}{2}' \\ \underline{62\ \ 15} \\ 40^\circ \text{ nearly.} \end{array}$$

Longitude = G.M.T. + E − H.A.

Sun's H.A. 20° 06′ W. West because azimuth west.

= 1 hr. 20 mins. 24 secs.

	hrs.	mins.	secs.	
G.M.T.	14	00	00	23–10–36
E	12	15	38	
	26	15	38	
H.A.	1	20	24	
	24	55	14	
	24			
Longitude		55	14	W.

∴ = 13° 48½′ W. West because formula time is less than 12 hrs. after deducting the 24 hrs. according to rule. (See pages 20–21.)

The estimated position is therefore Lat. 50° 00′ N., Long. 13° 48½′ W., which brings the position into a zone where the Magnetic Variation is not as much as 20° W. and so the azimuth angle at B is a few degrees out. Had the Magnetic Variation been 16° W. instead of 20° W. as taken, this would bring the position to be Lat. 49° N., Long. 10° 12′ W., from which it will be seen that unless an accurate angle or azimuth of a body has been obtained the resultant position is only very approximate. It does, however, give some indication of where one is, and enables the latitude to be established with more accuracy than the longitude.

APPENDIX I

LOGARITHMS

WHAT are logarithms? Surely it does not matter what they are so long as you know how to use them.

It is not generally known that every common or garden number has another number called a "logarithmic number," and yet it has. The logarithmic number of 3 is ·4771. Had the logarithmic number ·4771 been known, the ordinary number may be found to be 3. The way ordinary numbers are put into logarithmic numbers and vice versa is by means of logarithmic tables. In these tables the ordinary number is given down the left-hand column and across the top, the corresponding logarithmic number being found in the body of the tables.

Logarithms are used for multiplication or division. It is a quick way of doing sums. To multiply, turn the ordinary numbers you are going to multiply into logarithmic numbers. Merely add these logarithmic numbers together. Turn the sum of the logarithmic numbers back into an ordinary number and the sum is done. To divide, subtract the logarithmic numbers from each other instead of adding them.

There are two parts of a logarithm, (*a*) the "Characteristic," (*b*) the "Mantissa." The characteristic is always in front of the decimal point and may be positive or negative. The mantissa is always behind the decimal point and is always positive. It is also the actual logarithmic number of the ordinary number. The characteristic is merely a device for placing the number of digits in front of the decimal point. The ordinary number 30 may also be written 30·00 . . ., etc. The adding of 0's behind the decimal point does not alter its value. 30·0 has two digits in front of the decimal point.

Rule for Characteristic

The characteristic of the logarithmic number is always one less numerically than the number of digits in front of the decimal point. When there are no digits in front of the decimal point as in ·3, the characteristic is written $\bar{1}$ (bar 1). Also, when there are no digits in front of the decimal point and at the same time there are some 0's between the decimal point and the first figure, the characteristic is numerically greater by 1 than the number of 0's. Thus ·03 takes the characteristic $\bar{2}$·, ·003 the characteristic $\bar{3}$, etc. Remember, the

characteristic has nothing to do with the actual logarithmic number which deals solely with the ordinary number.

If we wish to multiply 1649 by 72·63, it would take more space and time by the long way. By logs it is very easily done.

$$\begin{aligned} \text{Log } 1649 &= 3{\cdot}217221 \\ \text{Log } 72{\cdot}63 &= \underline{1{\cdot}861116} \\ &\ 5{\cdot}078337 \end{aligned}$$

$$= 119790{\cdot}0\ldots.0$$

Here, 217221 is the mantissa, i.e. the logarithmic number of the ordinary number 1649. It has been found from the tables of logarithms. The 3 in front of 217221 is the characteristic for the ordinary number 1649, i.e. one less numerically than the number of digits in front of the decimal point. Actually there is no visible decimal point in the number 1649, but one could be inserted after the 9 without altering its value.

861116 is the logarithmic number of the ordinary number 7263. The 1 in front of 861116 means that the ordinary number has two digits in front of the decimal point.

The 119790·0 has been found by looking in the body of the Log tables and seeing what ordinary number has a logarithmic number of 078337. The 5 has nothing to do with the logarithmic number. It merely shows that the ordinary number must have 6 digits in front of the decimal point.

Concerning Numbers

Any whole number can have a decimal point put after it without altering its value, as has already been explained. Also, any number can be made into a nearer all round whole number.

Thus	2258	to	the	nearest	1
becomes	2260	,,	,,	,,	10
and	2300	,,	,,	,,	100
and	2000	,,	,,	,,	1000

When using logarithms or a slide rule, this principle has to be applied. If, after multiplying some figures which are known to have four figures in front of the decimal point and it comes to 225 something, you could write 2250 with sufficient accuracy. Therefore, the bigger the number in front of the decimal point the less need you worry about the last figure. Any number after the decimal point is always less than 1 or unity. Thus 22·58 would be 22·6 to the nearest first place of decimals, and 23·0 to the nearest whole

number. The figure 119790·0 in the previous example may not therefore be strictly accurate but will be sufficiently accurate for practical purposes.

Further Examples

1·649 × ·7263

$$\begin{aligned} \text{Log } 1{\cdot}649 &= 0{\cdot}217221 \\ \text{Log } {\cdot}7263 &= \bar{1}{\cdot}861116 \\ \hline & 0{\cdot}078337 \end{aligned}$$

Answer 1·1979. (One digit in front of the decimal point because the characteristic is 0.)

·01649 × ·007263

$$\begin{aligned} \text{Log } {\cdot}01649 &= \bar{2}{\cdot}217221 \\ \text{Log } {\cdot}007263 &= \bar{3}{\cdot}861116 \\ \hline & \bar{4}{\cdot}078337 \end{aligned}$$

Answer ·00011979. (Three 0's after the decimal point because the characteristic is $\bar{4}$.)

Division by Logarithms

Divide 1649 by 72·63

$$\begin{aligned} \text{Log } 1649 &= 3{\cdot}217221 \\ \text{Log } 72{\cdot}63 &= 1{\cdot}861116 \\ \hline & 1{\cdot}356105 \end{aligned}$$

Answer 22·704.

22704 is the number whose logarithmic number is 356105 and as the characteristic is 1 there must be two places in front of the decimal point. Hence the number is 22·704.

Divide 1·649 by ·7263

$$\begin{aligned} \text{Log } 1{\cdot}649 &= {\cdot}217221 \\ \text{Log } {\cdot}7263 &= \bar{1}{\cdot}861116 \\ \hline & 0{\cdot}356105 \end{aligned}$$

Answer 2·2704

Here the subtraction sum above rather complicates the result so far as the characteristic is concerned. In all subtraction sums of whatever kind what happens is that the sign of the number being subtracted is changed and then the quantities are added. Thus, if we subtract 4 from 9, what we really do is to add − 4 to + 9 and

retain the sign of the greater = + 5 or 5. If we subtract 9 from 4 we actually add − 9 to + 4, giving a result of −5.

In the case in point we are subtracting $\bar{1}{\cdot}861116$ from ·217221. Thus we are adding

$$\begin{array}{rcrl} -\ \bar{1}{\cdot}000000 & = & +\ 1{\cdot}000000 & \text{Two negatives make a positive} \\ -\ {\cdot}861116 & = & -\ \ {\cdot}861116 & \\ \hline & & +\ \ {\cdot}138884 & \end{array}$$

to

$+\ {\cdot}217221.$

$$\begin{array}{rl} + & {\cdot}138884 \\ + & {\cdot}217221 \\ \hline + & {\cdot}356105 \quad \text{or } 0{\cdot}356105. \end{array}$$

If one likes—and it is done in the case of logarithmic trigonometrical ratios—

Characteristic	0	may be called	10
,,	1	,,	11
,,	2	,,	12
,,	$\bar{1}$	,,	9
,,	$\bar{2}$	,,	8
,,	$\bar{3}$	,,	7

Calling the characteristic by these terms

$$\begin{array}{rl} \text{Log } 1{\cdot}649 = & 10{\cdot}217221 \\ \text{Log } {\cdot}7263 = & 9{\cdot}861116 \\ \hline & 0{\cdot}356105 = 2{\cdot}2704 \end{array}$$

Divide ·01649 by ·0007263

$$\begin{array}{rlcl} \text{Log } {\cdot}01649 = & \bar{2}{\cdot}217221 & \text{or} & 8{\cdot}217221 \\ \text{Log } {\cdot}0007263 = & \bar{4}{\cdot}861116 & \text{or} & 6{\cdot}861116 \\ \hline & 1{\cdot}356105 & & 1{\cdot}356105 \end{array}$$

$$= 22{\cdot}704$$

To find the square root or cube root, divide the logarithmic number by 2 or 3 respectively. The square or cube would be found by multiplying the logarithmic number by 2 or 3 respectively.

To find the square root of 64.

$$\text{Log } 64 = 1{\cdot}8062 \qquad \begin{array}{r|l} 2 & 1{\cdot}8062 \\ \hline & \ \ {\cdot}9031 \end{array}$$

The number whose Log is ·9031 is 8·0.

To find the cube root of 27.

$$\text{Log } 27 = 1{\cdot}4314 \qquad 3\underline{|1{\cdot}4314} \atop {\cdot}4771$$

The number whose Log is ·4771 is 3·0.

To find the square root of ·64.

$$\text{Log } {\cdot}64 = \bar{1}{\cdot}8062$$

Here the characteristic is negative and the mantissa 8062 is positive, and the whole has to be divided by 2. To do this, make the characteristic divisible by 2 and you get, without altering, the value of the whole number in front of the decimal point,

$$\bar{2} + 1{\cdot}8062 \qquad 2\underline{|\bar{2} + 1{\cdot}8062} \atop \bar{1} + {\cdot}9031 \quad \text{or } \bar{1}{\cdot}9031.$$

The number whose Log is $\bar{1}{\cdot}9031$ is ·8.

To find the cube root of 0·169

$$\text{Log } {\cdot}169 = \bar{1}{\cdot}2279 \quad \text{or } \bar{3} + 2{\cdot}2279$$

$$3\underline{|\bar{3} + 2{\cdot}2279} \atop \bar{1}{\cdot}7426 = {\cdot}553.$$

In these cases the characteristic of 9, 7, etc., must not be used as was permissible for ordinary division sums.

Trigonometrical Ratios

In any right-angled triangle, which is a triangle in which one of the angles is 90°,

The Sine—written Sin—of the angle is $\dfrac{\text{Opposite side}}{\text{Hypotenuse}}$

The Cosine ,, Cos ,, $\dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$

The Tangent ,, Tan ,, $\dfrac{\text{Opposite side}}{\text{Adjacent side}}$

The Cotangent ,, Cot ,, $\dfrac{\text{Adjacent side}}{\text{Opposite side}}$

The Secant ,, Sec ,, $\dfrac{\text{Hypotenuse}}{\text{Adjacent side}}$

The Cosecant ,, Cosec ,, $\dfrac{\text{Hypotenuse}}{\text{Opposite side}}$

These ratios may be expressed in the form of a fraction or as a decimal. It is usual to express them as a decimal. Thus, if BC is 1 and AB is 4, Tan A would be $\frac{1}{4}$, or ·25. This is called the Natural Tangent. The tables for Natural Tangents show that the angle whose Natural Tangent is ·25 is 14° to the nearest degree.

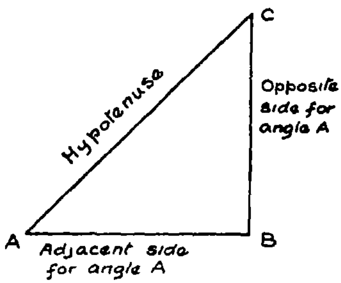


FIG. 20

Knowing the angle A to be 14°, the side AC may be found from the Cosine or Sine.

$$\frac{AB}{AC} = \text{Cos } A$$

$$\therefore\ AC = \frac{AB}{\text{Cos } A} = \frac{4}{\text{Cos } A}$$

The Natural Cosine of 14° = ·9703, so that $AC = \frac{4}{\cdot 9703}$

This would have to be worked out. It could be worked out by Logs.

$$\begin{array}{rrcr} \text{Log } 4 = & 0{\cdot}6021 & \text{or} & 10{\cdot}6021 \\ \text{Log } {\cdot}9703 = & \bar{1}{\cdot}9869 & \text{or} & 9{\cdot}9869 \\ \hline & 0{\cdot}6152 & & 0{\cdot}6152 \end{array}$$

The ordinary number whose Log is 6152 is 4123, but as the characteristic is 0 it will be 4·123.

$$\therefore \qquad AC = 4{\cdot}123$$

When it is known that multiplication or division is to be done the Log Sine, Log Cosine or Log Tangent can be found direct from the tables instead of the natural equivalent. This is merely the logarithm of the natural equivalent. Thus, had 4 to be divided by Cos 14°, it would be quicker to find the Log Cosine of 14° direct.

$$\begin{array}{lcl} \text{Log } 4 & = & 0{\cdot}6021 \quad \text{or} \quad 10{\cdot}6021 \\ \text{Log Cos } 14° & = & \underline{9{\cdot}9869} \quad \text{from tables for Log Cos } 14° \\ & & 0{\cdot}6152 \end{array}$$

. Tan 60° × Sin 40°

$$\begin{array}{lcr} \text{Log Tan } 60° & = & 10{\cdot}2386 \\ \text{Log Sin } 40° & = & \underline{9{\cdot}8081} \\ & & 20{\cdot}0467 \end{array}$$

When the characteristic is more than 10, discard 10. The resultant characteristic is therefore 10 in the above case, but a characteristic of 10 is 0, so that we want the ordinary number whose Log is 0·0467. It is 1·114.

Therefore Tan 60° × Sin 40° = 1·114.

Formulae for Solution of Plane Triangles other than Right-angled Triangles

Given two angles and one side, or two sides and one angle—

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

Given two sides and the included angle—

$$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$$

or

$$b^2 = c^2 + a^2 - 2ca \text{ Cos } B$$

or

$$c^2 = a^2 + b^2 - 2ab \text{ Cos } C$$

Given three sides—

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

or

$$\text{Cos } B = \frac{c^2 + a^2 - b^2}{2ca}$$

or

$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$

The sum of three angles in all plane triangles adds up to 180°, so that if two angles are known the other may be found.

Some of the trigonometrical tables in pocket books and the like do not give values for Secants and Cosecants. The following will enable this to be overcome—

$$\text{Sec } A = \frac{1}{\text{Cos } A} \qquad \therefore \text{ Sin } B \text{ Sec } A = \frac{\text{Sin } B}{\text{Cos } A}$$

$$\text{Cosec } A = \frac{1}{\text{Sin } A}$$

$$\text{Cot } A = \frac{1}{\text{Tan } A}$$

APPENDIX II

For use with the AIR ALMANAC

The Air Almanac introduced by the Air Ministry in October, 1937, is excellent for finding the Hour Angle of heavenly bodies, but it does not provide, at sight, certain quantities necessary for astronomical computations. These quantities may, however, be ascertained indirectly from the Air Almanac as follows—

Value E for Sun

Equation of Time = G.M.T. in 24 hr. system from noon as starting point $\sim$ G.H.A. sun.

If G.H.A. is more than G.M.T., then True Sun is clockwise of Mean Sun, and if it is clockwise *E* is 12 hr. + Equation of Time.

Value R for Sun

R = G.H.A.*r* – 180° at noon G.M.T.
– 270° at 18 hr. G.M.T.
– 360° at mid-night G.M.T.
– 90° at 6 hr. G.M.T.

R.A.M.

R.A.M. = G.H.A.*r* – *W* or + *E* Longitude.

R.A. Moon

R.A. Moon = (360° – G.H.A. moon) + G.H.A.*r*

Time to Arc

TIME		ARC
A Hrs.	=	*B*°
C Min.	=	*D*°
A Min.	=	*B*′
C Sec.	=	*D*′

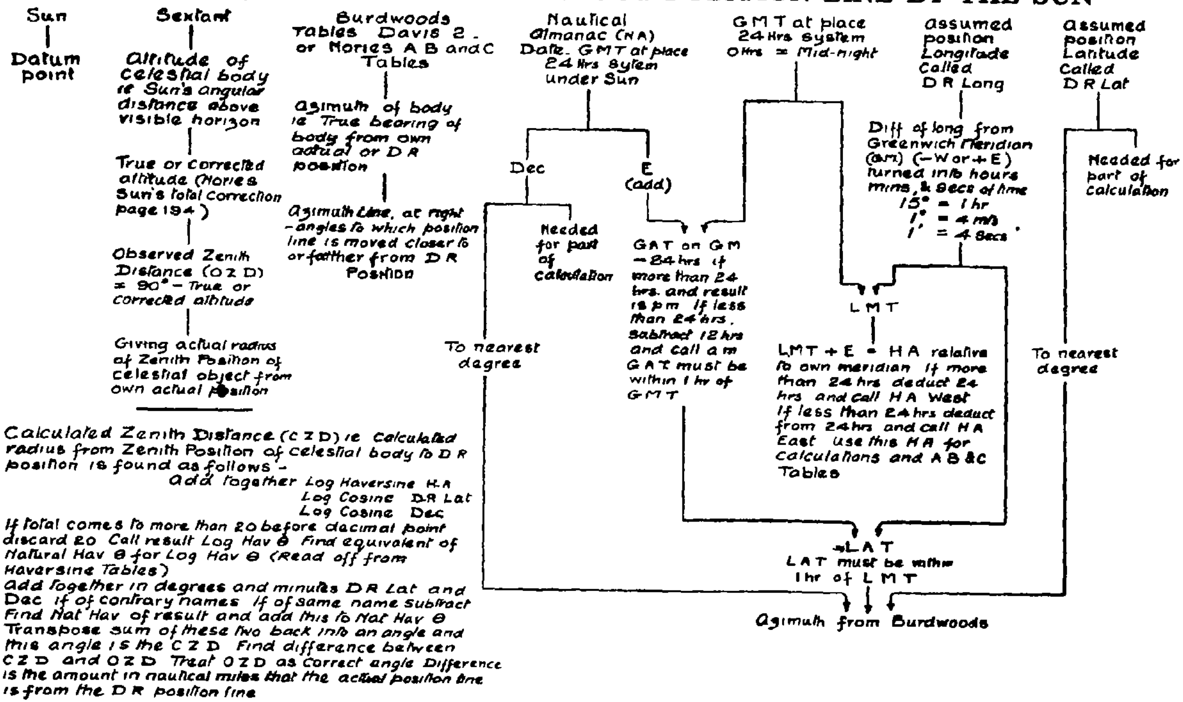
A	*B*	*A*	*B*
1	15	9	135
2	30	10	150
3	45	11	165
4	60	12	180
5	75		
6	90		
7	105		
8	120		

Arc to Time

ARC		TIME
D°	=	*C* Min.
B°	=	*A* Hrs.
B′	=	*A* Min.
D′	=	*C* Sec.

C	*D*	*C*	*D*
4	1	36	9
8	2	40	10
12	3	44	11
16	4	48	12
20	5	52	13
24	6	56	14
28	7	60	15
32	8		

DATA REQUIRED FOR WORKING OUT POSITION LINE BY THE SUN



Calculated Zenith Distance (C Z D) ie Calculated radius from Zenith Position of celestial body to D R position is found as follows –
Add together Log Haversine H.A
Log Cosine D.R Lat
Log Cosine Dec
If total comes to more than 20 before decimal point discard 20 Call result Log Hav θ Find equivalent of Natural Hav θ for Log Hav θ (Read off from Haversine Tables)
Add together in degrees and minutes D R Lat and Dec if of contrary names If of same name subtract Find Nat Hav of result and add this to Nat Hav θ Transpose sum of these two back into an angle and this angle is the C Z D Find difference between C Z D and O Z D Treat O Z D as correct angle Difference is the amount in nautical miles that the actual position line is from the D R position line

DATA REQUIRED FOR WORKING OUT POSITION LINE BY STAR, PLANET OR MOON

1 Datum point.

2 Altitude, thence a.z D

3 Burdwoods Tables or A B & C Tables

4 Calculated Zenith Distance C Z D

Same procedure as for Sun

Dec = Declination

G.A.T = Greenwich Apparent time

L.A.T = Local "

R.A. = Right Ascension

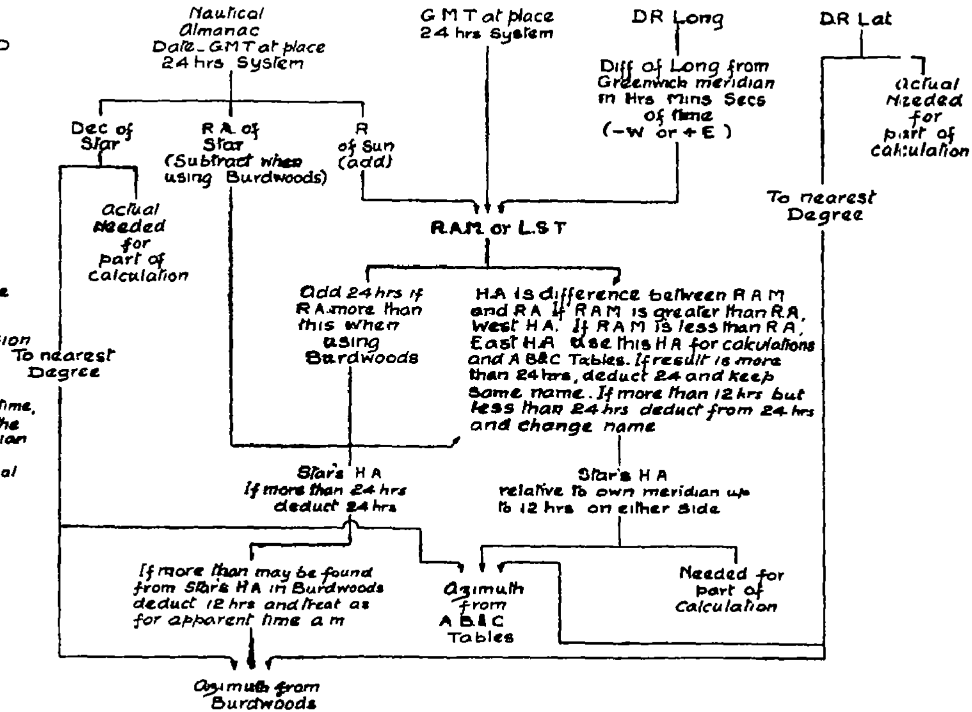
R = R.A. of Mean Sun ± 12 hrs

E = Equation of time, 12 hrs minus the

R.A.M. = R A of meridian

L.S.T = Local Sidereal time.

H A = Hour Angle



EPILOGUE

THE following extract appeared in *The Times* on the 4th June, 1937—

THE SUN AS A SIGNPOST

NEW FLYING USE FOR SEXTANT SIGHTS

From Our Aeronautical Correspondent

The familiar navigational routine of "shooting the sun" with a sextant for the purpose of calculating the position of the observer on the earth's surface seems likely to be given a new application on comparatively short flights in which the weather makes map-reading impossible.

Mr. G. W. Ferguson, a charter pilot, has just related for the benefit of members of the Guild of Air Pilots how he got himself with Coronation films from Heston to Manchester on Coronation Day and afterwards made the return flight with the aid of a bubble sextant, a compass, and a clock. The interesting part of the adventure is that he relied on a sextant sight for his signal to descend through the clouds close to his destinations.

His method was to work out in advance a series of sun altitudes at 15-minute intervals for Manchester. He then took off, measured his angle of drift, went up through the low clouds and continued on the appropriate course until the clock and the angle of altitude shown on his sextant combined to declare that he must be over the Manchester airport. From a height of 5000 ft. he then came down through the clouds and found himself near the Manchester Ship Canal and about five miles from the aerodrome. On the return journey his method was slightly less successful, but it brought him within sight of the ground near Slough some eight miles from Heston.

"The conclusion I arrived at," said Mr. Ferguson, "was that it is possible to get a rough idea as to whether you have reached a place by means of a sextant and pre-calculated altitudes."

So far as is known, the trip referred to is the first occasion in England on which a civil air pilot, flying solo, without radio, and with a completely overcast sky with a very low cloud base, has used a sextant to know when he has reached his objective. Advantage of the occasion was taken by the Author to test, under the worst possible working conditions, some of the lessons to be learnt from this book, and it may be of interest to record that several pilots deemed it wiser—and rightly so—to cancel their projected flights owing to the appalling visibility appertaining throughout central and eastern England on this particular day.

INDEX